Reading Quiz

1. The type of function that describes simple harmonic motion is

   A. linear
   B. exponential
   C. quadratic
   D. sinusoidal
   E. inverse

Answer: D

These reading quiz questions are a nice way to get the students minds into the game at the start of class. They are fairly straight forward.

Answer

1. The type of function that describes simple harmonic motion is

   A. linear
   B. exponential
   C. quadratic
   D. sinusoidal
   E. inverse

Answer: D

Reading Quiz

2. A mass is bobbing up and down on a spring. If you increase the amplitude of the motion, how does this affect the time for one oscillation?

   A. The time increases.
   B. The time decreases.
   C. The time does not change.

Answer: C

Answer

2. A mass is bobbing up and down on a spring. If you increase the amplitude of the motion, how does this affect the time for one oscillation?

   A. The time increases.
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   C. The time does not change.

Answer: C

Reading Quiz

3. If you drive an oscillator, it will have the largest amplitude if you drive it at its ________ frequency.

   A. special
   B. positive
   C. natural
   D. damped
   E. pendulum

Answer: C

Answer

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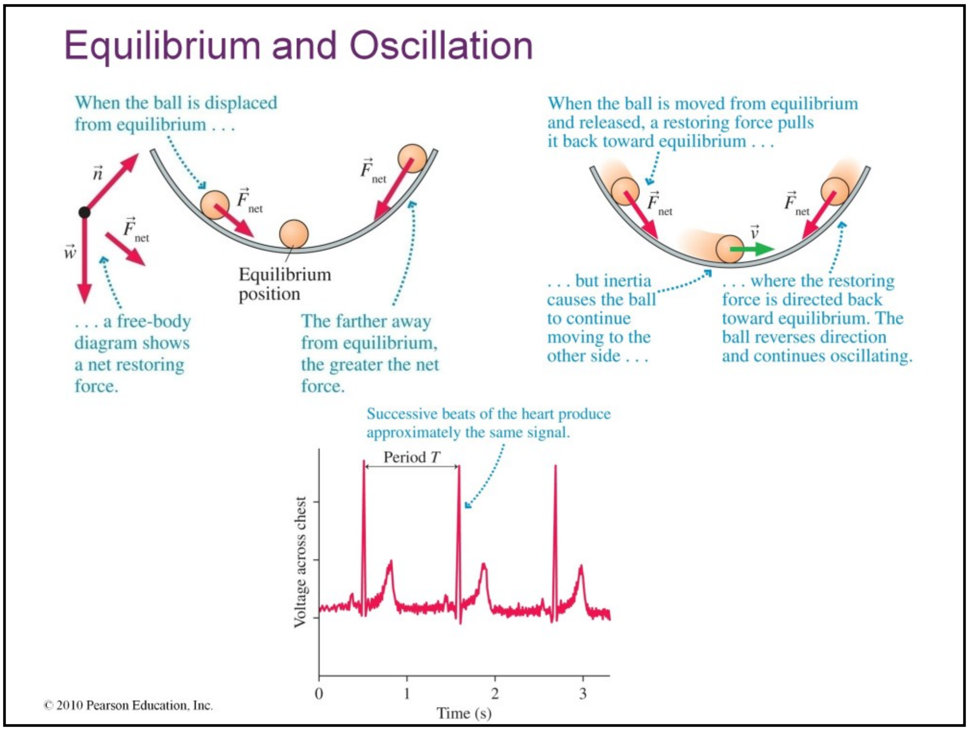
Answer: C

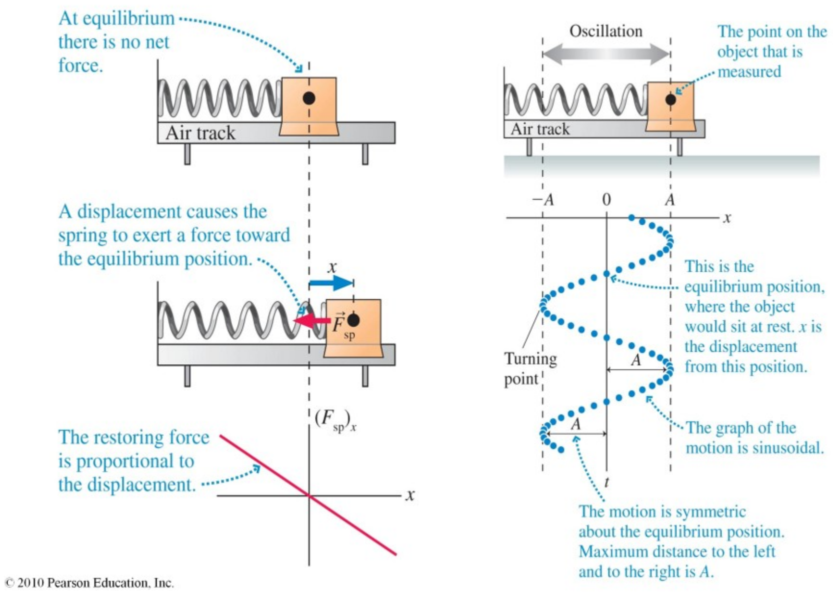


I used this to start a discussion about if an EKG is Simple Harmonic Motion (SHM). First I asked if it could be and let the students discuss with their neighbor. Then we had a whole class discussion. It is not because there's not a linear restoring force that causes the heart to beat. You can see this because it's not a sinusoidal function. The ball in a bowl is however, a nice example.

Linear Restoring Forces and Simple Harmonic Motion

- Simple harmonic motion (SHM) is the motion of an object subject to a force that is proportional to the object's displacement. One example of SHM is the motion of a mass attached to a spring. In this case, the relationship between the spring force and the displacement is given by Hooke's Law, $F = -kx$, where k is the spring constant, x is the displacement from the equilibrium length of the spring, and the minus sign indicates that the force opposes the displacement.

Linear Restoring Forces and Simple Harmonic Motion

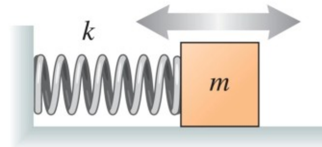


Frequency and Period

The frequency of oscillation depends on physical properties of the oscillator; it does not depend on the amplitude of the oscillation.

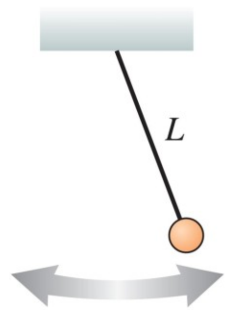
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{and} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

Frequency and period of SHM for mass m on a spring with spring constant k



$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \quad \text{and} \quad T = 2\pi\sqrt{\frac{L}{g}}$$

Frequency of a pendulum of length L with free-fall acceleration g



Sinusoidal Relationships

Sinusoidal relationships

A quantity that oscillates in time and can be written

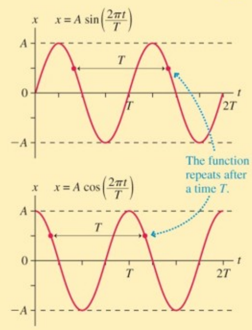
$$x = A\sin\left(\frac{2\pi t}{T}\right)$$

or

$$x = A\cos\left(\frac{2\pi t}{T}\right)$$

is called a **sinusoidal function** with **period** T. The argument of the functions, $2\pi t/T$, is in radians.

The graphs of both functions have the same shape, but they have different initial values at $t = 0$ s.



LIMITS If x is a sinusoidal function, then x is:

- *Bounded*—it can take only values between A and $-A$.
- *Periodic*—it repeats the same sequence of values over and over again. Whatever value x has at time t, it has the same value at $t + T$.

SPECIAL VALUES The function x has special values at certain times:

	$t = 0$	$t = \frac{1}{4}T$	$t = \frac{1}{2}T$	$t = \frac{3}{4}T$	$t = T$
$x = A\sin(2\pi t/T)$	0	A	0	$-A$	0
$x = A\cos(2\pi t/T)$	A	0	$-A$	0	A

Exercise 6

This page demonstrates how the position equation can be sin or cos. It simply depends on the initial amplitude which means it just depends on when you start time.

Each dimension of circular motion is a sinusoidal funtion.

- Animation of sine and cosine

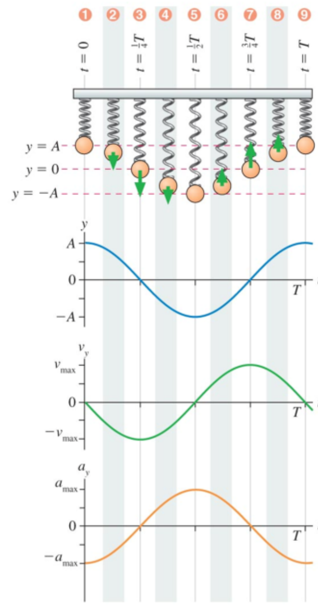
Mathematical Description of Simple Harmonic Motion

$$x(t) = A\cos(2\pi f t)$$

$$v_x(t) = -(2\pi f)A\sin(2\pi f t)$$

$$a_x(t) = -(2\pi f)^2 A\cos(2\pi f t)$$

Position, velocity, and acceleration for an object in simple harmonic motion with frequency f and amplitude A



Students have a surprisingly hard time with many aspects of the math of Acos(2pift) The next series of questions helps with some of it. I also used the workbooks from Knight, Jones and Fields in recitation and the work for this chapter really helped students sort out how the math works here.

$$x(t) = A\cos(2\pi ft)$$

$$v_x(t) = -(2\pi f)A\sin(2\pi ft)$$

$$a_x(t) = -(2\pi f)^2 A\cos(2\pi ft)$$

Position, velocity, and acceleration for an object in simple harmonic motion with frequency f and amplitude A

What is the largest value $\cos(2\pi ft)$ can ever have?

A. 0
B. 1
C. $2\pi ft$
D. ∞
E. ½

$$x(t) = A\cos(2\pi ft)$$

$$v_x(t) = -(2\pi f)A\sin(2\pi ft)$$

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A. 0
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Maximum displacement, velocity and acceleration

$$x(t) = A\cos(2\pi ft)$$
$$v_x(t) = -(2\pi f)A\sin(2\pi ft)$$
$$a_x(t) = -(2\pi f)^2 A\cos(2\pi ft)$$

What is the largest displacement possible?

A. 0
B. A
C. $2\pi ft$
D. ∞
E. ½ A

Maximum displacement, velocity and acceleration

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$$v_x(t) = -(2\pi f)A\sin(2\pi ft)$$
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What is the largest velocity possible?

A. 0
B. $2\pi f$
C. $2A\pi f$
D. $A\pi f$
E. ∞

Maximum displacement, velocity and acceleration

$$x(t) = A\cos(2\pi ft)$$
$$v_x(t) = -(2\pi f)A\sin(2\pi ft)$$
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What is the largest velocity possible?

A. 0
B. $2\pi f$
C. **$2A\pi f$**
D. $A\pi f$
E. ∞

Maximum displacement, velocity and acceleration

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$$v_x(t) = -(2\pi f)A\sin(2\pi ft)$$
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What is the largest acceleration possible?

A. 0
B. $2A(\pi f)^2$
C. $4\pi^2 A f^2$
D. ∞
E. $2A\pi f$

Maximum displacement, velocity and acceleration

$$x(t) = A\cos(2\pi f t)$$
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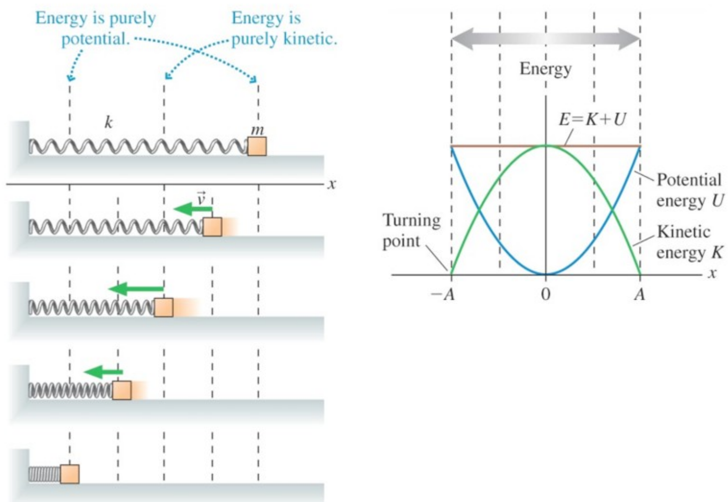
A. 0
B. $2A(\pi f)^2$
C. $4\pi^2 A f^2$
D. ∞
E. $2A\pi f$

Energy in Simple Harmonic Motion

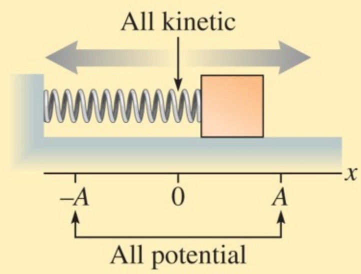
As a mass on a spring goes through its cycle of oscillation, energy is transformed from potential to kinetic and back to potential.



Energy

If there is no friction or dissipation, kinetic and potential energies are alternately transformed into each other in SHM, with the sum of the two conserved.

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$$
$$= \frac{1}{2}mv_{\text{max}}^2$$
$$= \frac{1}{2}kA^2$$



Solving Problems

TACTICS BOX 14.1 **Identifying and analyzing simple harmonic motion**

1. If the net force acting on a particle is a linear restoring force, the motion is simple harmonic motion around the equilibrium position.
2. The position, velocity, and acceleration as a function of time are given in Equations 14.18. The equations are given here in terms of x, but they can be written in terms of y, θ, or some other variable if the situation calls for it.
3. The amplitude A is the maximum value of the displacement from equilibrium. The maximum speed and the maximum magnitude of the acceleration are $v_{max} = 2\pi f A$ and $a_{max} = (2\pi f)^2 A$.
4. The frequency f (and hence the period $T = 1/f$) depends on the physical properties of the particular oscillator, but f does *not* depend on A.

   For a mass on a spring, the frequency is given by $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.
5. The sum of potential energy plus kinetic energy is constant. As the oscillation proceeds, energy is transformed from kinetic into potential energy and then back again.

$$x(t) = A\cos(2\pi f t)$$

$$v_x(t) = -(2\pi f)A\sin(2\pi f t)$$

$$a_x(t) = -(2\pi f)^2 A\cos(2\pi f t)$$

Equations

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{and} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

Frequency and period of SHM
for mass m on a spring with spring constant k

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \quad \text{and} \quad T = 2\pi\sqrt{\frac{L}{g}}$$

Frequency of a pendulum of length L with free-fall acceleration g

$$x(t) = A\cos(2\pi ft)$$

$$v_x(t) = -(2\pi f)A\sin(2\pi ft)$$

$$a_x(t) = -(2\pi f)^2 A\cos(2\pi ft)$$

$$v_{max} = 2\pi fA \text{ and } a_{max} = (2\pi f)^2 A.$$

$$\begin{aligned} E &= \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 \\ &= \frac{1}{2}mv_{max}{}^2 \\ &= \frac{1}{2}kA^2 \end{aligned}$$

Additional Example Problem

Walter has a summer job babysitting an 18 kg youngster. He takes his young charge to the playground, where the boy immediately runs to the swings. The seat of the swing the boy chooses hangs down 2.5 m below the top bar. “Push me,” the boy shouts, and Walter obliges. He gives the boy one small shove for each period of the swing, in order keep him going. Walter earns $6 per hour. While pushing, he has time for his mind to wander, so he decides to compute how much he is paid per push. How much does Walter earn for each push of the swing?

I let the class try this problem out while the equations were showing. Then after they had worked for a reasonable amount of time I got ideas from them how to solve the problem and then did it on the board.

Additional Example Problem

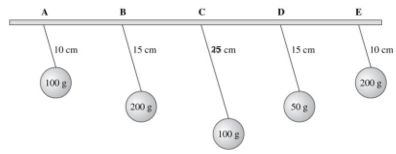
Walter has a summer job babysitting an 18 kg youngster. He takes his young charge to the playground, where the boy immediately runs to the swings. The seat of the swing the boy chooses hangs down 2.5 m below the top bar. "Push me," the boy shouts, and Walter obliges. He gives the boy one small shove for each period of the swing, in order keep him going. Walter earns $6 per hour. While pushing, he has time for his mind to wander, so he decides to compute how much he is paid per push. How much does Walter earn for each push of the swing?

$T = 2\pi \sqrt{(l/g)} = 3.17$ s
3600 s/hr (1 push/3.17s) = 1136 pushes/ hr
$6/hr (1hr/1136 pushes) = $0.0053 / push

Or half a penny per push!

Checking Understanding

A series of pendulums with different length strings and different masses is shown below. Each pendulum is pulled to the side by the same (small) angle, the pendulums are released, and they begin to swing from side to side.



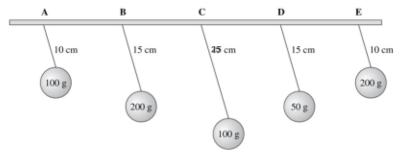
Rank the frequencies of the five pendulums, from highest to lowest.

A. $A = E > B = D > C$
B. $D > A = C > B = E$
C. $A = B = C = D = E$
D. $B > E > C > A > D$

Answer: A

Answer

A series of pendulums with different length strings and different masses is shown below. Each pendulum is pulled to the side by the same (small) angle, the pendulums are released, and they begin to swing from side to side.



Rank the frequencies of the five pendulums, from highest to lowest.

A. **A = E > B = D > C**
B. D > A = C > B = E
C. A = B = C = D = E
D. B > E > C > A > D

$f = 1/2\pi \sqrt{(g/l)}$
Frequency is proportional to the inverse of the length. So longer has a lower frequency

Answer: A

Example Problem

We think of butterflies and moths as gently fluttering their wings, but this is not always the case. Tomato hornworms turn into remarkable moths called hawkmoths whose flight resembles that of a hummingbird. To a good approximation, the wings move with simple harmonic motion with a very high frequency—about 26 Hz, a high enough frequency to generate an audible tone. The tips of the wings move up and down by about 5.0 cm from their central position during one cycle. Given these numbers,

A. What is the maximum velocity of the tip of a hawkmoth wing?

B. What is the maximum acceleration of the tip of a hawkmoth wing?

Again, let the class work on this in small groups and then did on the board.

Example Problem

We think of butterflies and moths as gently fluttering their wings, but this is not always the case. Tomato hornworms turn into remarkable moths called hawkmoths whose flight resembles that of a hummingbird. To a good approximation, the wings move with simple harmonic motion with a very high frequency—about 26 Hz, a high enough frequency to generate an audible tone. The tips of the wings move up and down by about 5.0 cm from their central position during one cycle. Given these numbers,

A. What is the maximum velocity of the tip of a hawkmoth wing?

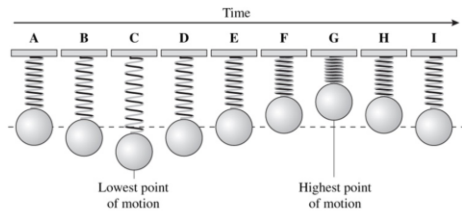
B. What is the maximum acceleration of the tip of a hawkmoth wing?

$v_{max} = 2\pi f A$ = 2 π 26Hz 0.05m = 8.17 m/s

$a_{max} = A\,(2\pi f)^2 =$ (2 π 26Hz)^2 0.05m = 1334 m/s^2

Example Problem

A ball on a spring is pulled down and then released. Its subsequent motion appears as follows:



1. At which of the above times is the displacement zero?
2. At which of the above times is the velocity zero?
3. At which of the above times is the acceleration zero?
4. At which of the above times is the kinetic energy a maximum?
5. At which of the above times is the potential energy a maximum?
6. At which of the above times is kinetic energy being transformed to potential energy?
7. At which of the above times is potential energy being transformed to kinetic energy?

Students have a hard time with several of these. The next series of slides breaks each into its own question. It took a good 15 minutes or more of class time to work through the ideas in their groups and as a whole class but it seemed to work well.

Answer:

1) A, E, I

2) C, G

3) A, E, I

4) A, E, I

5) C, G

6) B, F

7) D, H

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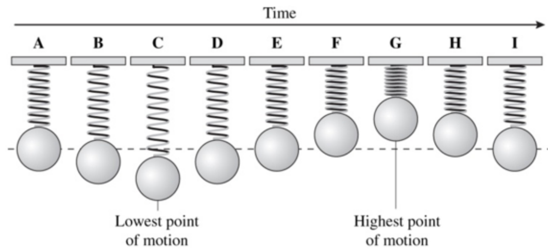

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D. D, H

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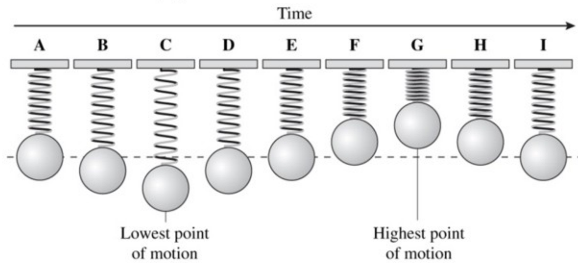

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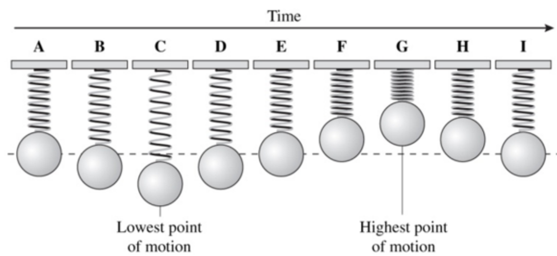

At which of the above times is the velocity zero?

A. C, G
B. A, E, I
C. B, F
D. D, H

Answer:

1) A, E, I

2) C, G

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7) D, H

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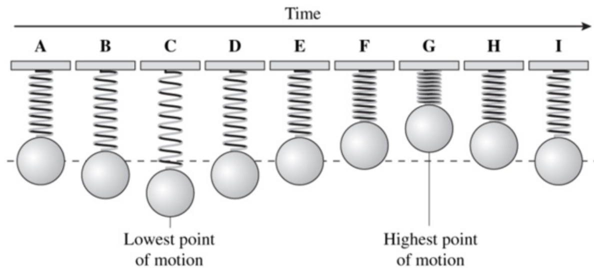

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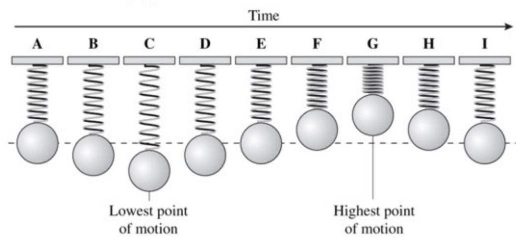


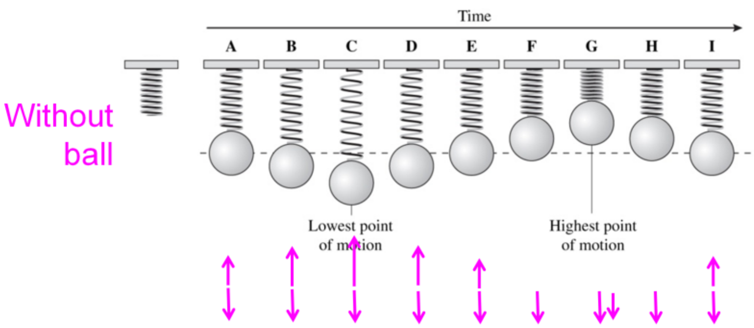
At which of the above times is the acceleration zero?

A. C, G
B. A, E, I
C. B, F
D. D, H

This one gave them the most difficulty so we looked at the free body diagram for each (on the next slide) to make sense out of it.

Example Problem

A ball on a spring is pulled down and then released. Its subsequent motion appears as follows:



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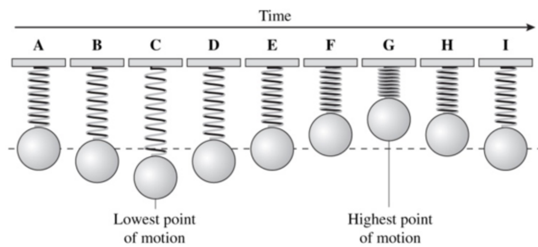


At which of the above times is the kinetic energy a maximum?

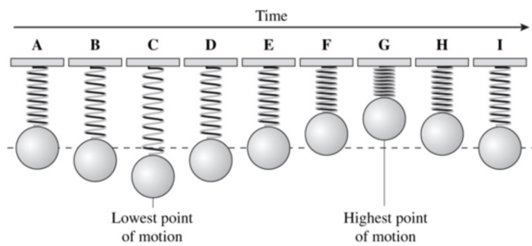
A. C, G
B. A, E, I
C. B, F
D. D, H

Answer:

1) A, E, I
2) C, G
3) A, E, I
4) A, E, I
5) C, G
6) B, F
7) D, H

Example Problem

A ball on a spring is pulled down and then released. Its subsequent motion appears as follows:



At which of the above times is the kinetic energy a maximum?

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D. D, H

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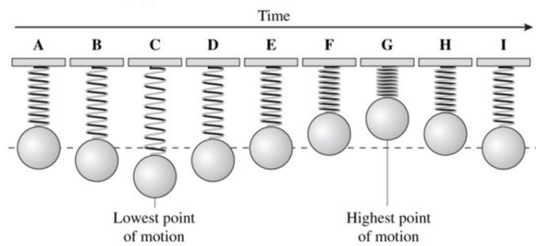

At which of the above times is the potential energy a maximum?

A. C, G
B. A, E, I
C. B, F
D. D. H

Example Problem

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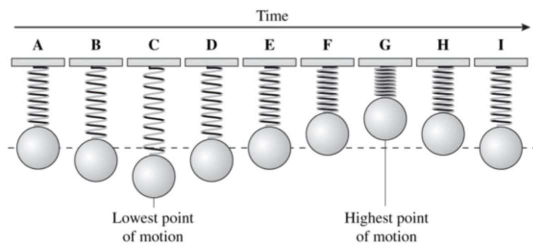

At which of the above times is the potential energy a maximum?

A. C, G
B. A, E, I
C. B, F
D. D, H

Example Problem

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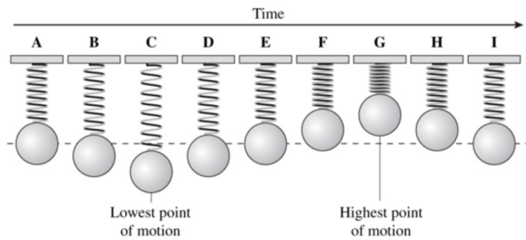
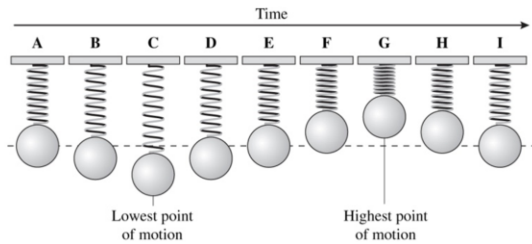


At which of the above times is kinetic energy being transformed to potential energy?

A. C, G
B. A, E, I
C. B, F
D. D, H

Example Problem

A ball on a spring is pulled down and then released. Its subsequent motion appears as follows:

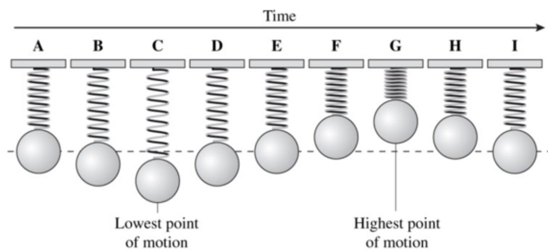


At which of the above times is kinetic energy being transformed to potential energy?

A. C, G
B. A, E, I
C. B, F
D. D, H

Example Problem

A ball on a spring is pulled down and then released. Its subsequent motion appears as follows:

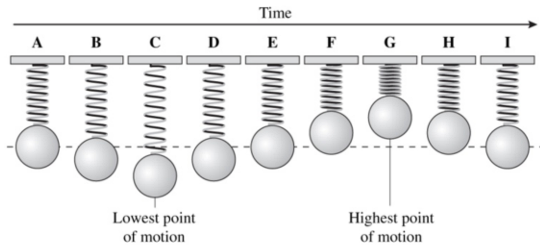


At which of the above times is potential energy being transformed to kinetic energy?

A. C, G
B. A, E, I
C. B, F
D. D, H

Example Problem

A ball on a spring is pulled down and then released. Its subsequent motion appears as follows:

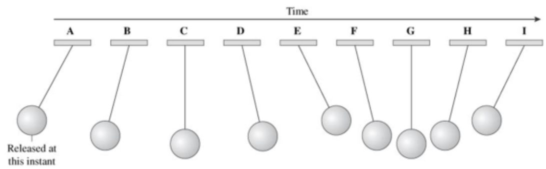


At which of the above times is potential energy being transformed to kinetic energy?

A. C, G
B. A, E, I
C. B, F
D. D, H

Example Problem

A pendulum is pulled to the side and released. Its subsequent motion appears as follows:



1. At which of the above times is the displacement zero?
2. At which of the above times is the velocity zero?
3. At which of the above times is the acceleration zero?
4. At which of the above times is the kinetic energy a maximum?
5. At which of the above times is the potential energy a maximum?
6. At which of the above times is kinetic energy being transformed to potential energy?
7. At which of the above times is potential energy being transformed to kinetic energy?

Answer:

1) C, G

2) A, E, I

3) C, G

4) C, G

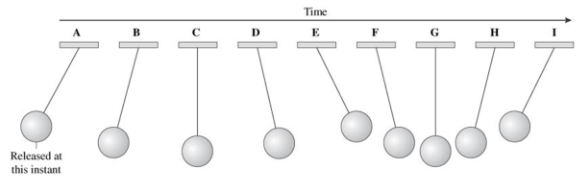
5) A, E, I

6) D, H

7) B, F

Example Problem

A pendulum is pulled to the side and released. Its subsequent motion appears as follows:



1. At which of the above times is the displacement zero? C, G
2. At which of the above times is the velocity zero? A, E, I
3. At which of the above times is the acceleration zero? C, G
4. At which of the above times is the kinetic energy a maximum? C, G
5. At which of the above times is the potential energy a maximum? A, E, I
6. At which of the above times is kinetic energy being transformed to potential energy? D, H
7. At which of the above times is potential energy being transformed to kinetic energy? B, F

Answer:

1) C, G

2) A, E, I

3) C, G

4) C, G

5) A, E, I

6) D, H

7) B, F

Great video showing how the bottom of a slinky does not fall until the compression has relaxed – very counterintuitive!!

Prediction

Why is the slinky stretched more at the top than the bottom?

A. The slinky is probably not in good shape and has been overstretched at the top.

B. The slinky could be designed that way and if he holds it the other way around, it'll be denser at the top than the bottom.

C. There's more mass pulling on the top loop so it has to stretch more to have a upward force kx equal to the weight mg.

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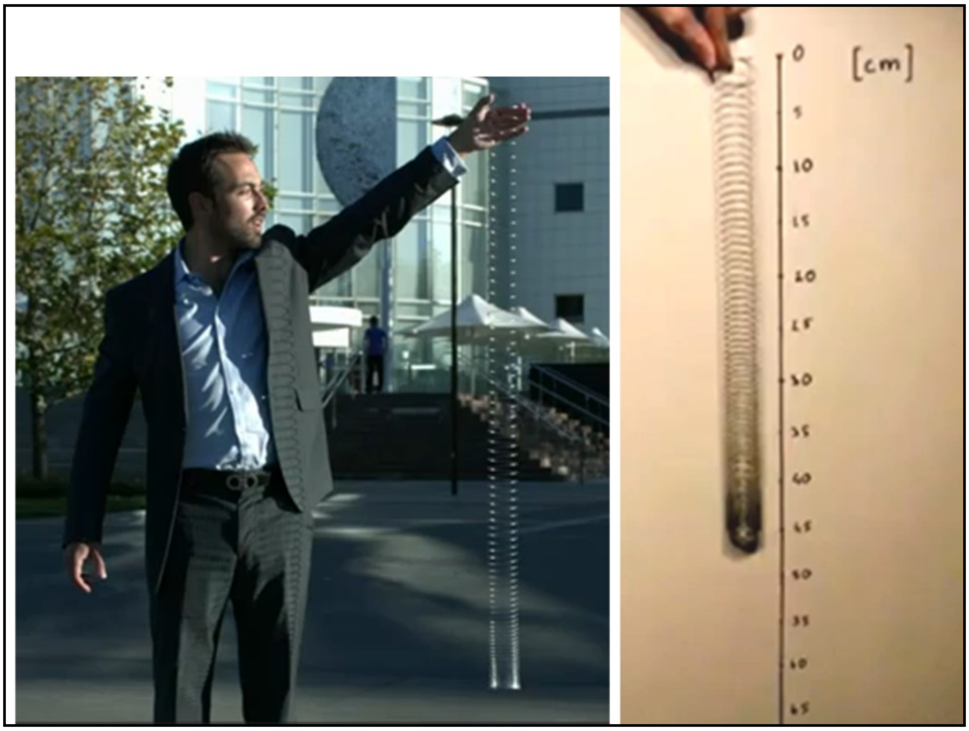
[cm]
0
5
10

Prediction

How will the slinky fall?

A. The entire thing will fall to the ground w/ an acceleration of g.

B. The center of mass will fall w/ an acceleration of g as the slinky compresses making it look like the bottom doesn't fall as fast.

C. The bottom will hover stationary in the air as the top falls until the top hits the bottom.

D. Other

watch video

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D. Other

Why does this happen?

murfleblurg 1 month ago

He explains this terribly. Each point of the spring is suspended only from the next point above it, and each point exactly matches the gravitational acceleration of the mass below it with upward elastic strain. The strain must be relaxed at each point before the strain in the point below it can relax, and it relaxes only as the coil collapses. The coil above must be collapsed in order to stop pulling up on the suspended spring. But strain isn't localized to points so relaxation acts as a wave.

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