Chapter I

Literature Review

First I will discuss previous work that's been done using the *knowledge and cognitive process* framework defined in Table I above. This will include a discussion of different approaches for studying problem solving as well as the uses and limitations of these approaches. The reader may notice an unequal amount of attention is focused on each problem category. This is not to be construed as the weighting of the value of each of these types of skills. This occurred because it is a literature review and thus is a rough reflection of the volume of work done in each area with the main focus being physics literature. There is such a vast amount of literature on acquisition of knowledge and some on beliefs within physics; however, processing research has been studied almost completely outside of physics, so a full coverage of it is beyond the scope of a dissertation. As an example strategies has a very large section and a substantial fraction of problem solving research – most of the research in physics in fact – has focused on strategy. In my opinion strategies are not any more important than facts and concepts, planning/monitoring or motivation. Instead this weighting is indicative of the very limited informal definition of a problem that has been used by physicist - similar to the typical back of the chapter task. Another category that does not get fair attention in this review is beliefs. In this case the fairly extensive attention that it received in my literature review still does not do it justice. There are many studies that do not consider the ideas of beliefs, expectations and motivation. Yet, beliefs about self, the environment, the problem and the discipline always affect the problem outcome. We are social creatures by nature – as much as a scientist, who likes clean experiments, may strive to remove such ‘difficulties’ our feelings about ourselves and what we are doing always play a role in how we behave.

Problem Solving Skills

The organization in Table I not only divides problem-solving skills into three major divisions, but it also represents how each division interacts with the others while a person solves problems. The divisions are made based on what problem solvers bring with them to the problem – knowledge – and what they do – processes with beliefs, expectations and motivation in between. Beliefs, expectations and their motivation are something that a person brings with them and has a different sense than other knowledge pieces. Mayer’s structure of “knowledge and processes” or “knowing and doing” covers all the skills needed to solve any sort of problem and does not imply significance to one over the other. Both are crucial to solving and the division helps to clarify things such as metacognitive knowledge and meta-processing which often get lumped together but have much different impacts on problem solving so it’s useful to create a clear separation.

The table also connects the specific types of knowledge used while engaging with different processes: When formulating a mental representation of the problem, solvers use the facts and concepts that they believe apply to the problem; While planning and monitoring the progress of the plan, solvers use problem solving strategies; and, when executing the plan, solvers use the procedural knowledge that they've learned applies to that specific topic. Beliefs & expectations is between knowledge and processes because they shape all of a student's cognitive processes. Beliefs, expectations and motivation mediate how and which knowledge items will be used. The one connection that the chart's organization doesn't relate is that procedural knowledge is specific to the topic so fits in with facts and concepts in this respect. All problem solving skills are interleaved in a somewhat complicated fashion; however, providing a clear framework helps one think about the specific instances of failure and provides a language for thinking and discussing problem solving.

Other researchers have used different categorization schemes for problem solving. Most of these schemes are quite similar but I have not found one as complete. As an example, Schoenfeld (1985) framed things only slightly differently using *resources*, *heuristics*, *control* and *beliefs*. Schoenfeld's "resources" encompass *facts*, *concepts* and *procedures,* which are all things that a solver has learned about the topic of the problem that need to be applied to solve the problem. Without these resources, it is unlikely the solver could produce any solution other than a very general one. Hueristics are strategies to help make the most of resources. The idea of Control – locating appropriate information, planning, monitoring, and meta-processing - includes decisions that determine the efficiency with which knowledge (resources and heuristics) is exploited. Schoenfeld's framing is useful but some skills, most notably executing which is *doing,* does not fit within his four categories. His framing also implies equal significance to heuristics and control which I believe is very misleading and maybe not his intention but the structure of having four categories lends itself to this assumption.

Knowledge

This section includes an overview of various studies on knowledge; however, the field is so extensive, a review of all work on this topic would be a book in itself. Because knowledge is such a key feature to solving problems, the bulk of problem solving literature and much of cognitive science literature focuses on how students acquire knowledge. Knowledge is composed of many specific components which I have listed in Table I. Facts are pieces of knowledge needed to solve the problem, concepts are an understanding of the ideas behind the problem. If it's a mathematical problem, a formula is a fact but the physical situation that the formula describes is a concept. Procedures are moves that are valid within the topic area such as algebraic manipulation. Strategies are more general than procedures, they are methods for solving problems such as considering a similar problem with fewer variables or breaking the problem into sub-goals.

The distinction between facts and concepts versus procedures is somewhat loose in physics; however in education a critical distinction is recognized between declarative and procedural memories (Cohen & Squire, 2004). Declarative memory consists of factual and conceptual information – "knowing what". Procedural memory is "knowing how". Declarative memory includes memory of example problems, formula or a definition and is usually quickly and easily learned. Procedural memory consists of a procedure, actual process of doing something, but not the factual information of how the procedure is done (that would be declarative). Tying your shoe is an example. Most of us cannot describe how to tie a shoe without either doing it or imagining doing it. The evidence for this distinction includes a large number of behavior studies and neuroscience studies showing differences in brain regions and amnesias (Anderson 1976 & 1998; Cohen & Eichenbaum ,1993; Eichenbaum & Cohen, 2004).

FACTS AND CONCEPTS

Facts are the basic bits and pieces of information that a person brings to bear when solving a problem. The level at which they understand these facts ("certain", "possible", "no idea") can shape the entire problem solving process. Conceptual understanding has no meaning without facts to build relations and ideas on. This type of knowledge is often referred to as declarative – "knowing what".

Classic Studies – i.e. problem classification

What solvers know as well as how it is organized and accessed are central issues in physics and cognitive psychology. The classic paper by Chi, Feltovich and Glaser (1981) described less experienced problem solvers as using surface features such as incline planes or pulleys to classify problems while graduate students and faculty members used deep structure or the concepts and procedures that would be used when solving the problems. This paper contained four studies, the largest of which had eight introductory students and eight graduate students while the other three had two to four total participants. The largest study was limited to having the subjects categorize the problems any way they want while the additional studies had slightly different directions such as name everything you can about these problems in three minutes. The subjects were not asked to solve problems in any of the four studies. The problems were specially selected or written to have surface features that did not typically go with the solution procedures. The authors believe that the differences seen could be an indication of differences in knowledge and possibly knowledge structure for these two types of problem solvers. In 2005 this paper had been referenced more than any other paper from *Cognitive Science.*

This study has been replicated many different times in various ways. Veldhuis (1986) replicated and extended the Chi study using 94 novices, five intermediate subjects and five experts. In his study the experts sorted based on deep structure but the novices sorted on either surface features or deep structure. More recently Sing (2007) also replicated the Chi et al study and found novices did not do as well as the experts; but, did do a better job of using deep structure than did Chi et al's students. In each case, Veldhuis and Sing, the students did better than did Chi et al's; however, the problems were not specially selected/written as they were in Chi's study. Schoenfeld (1985) also has had students sort problems finding that students before his intensive problem solving workshop are not as good at sorting based on deep structure. His specific instructions to the students were to create groups that are "similar mathematically in that they'd be solved in the same way"

Ferguson-Hessler and de Jong (1987) report on a study of knowledge structure of good versus poor students. Their research was done by having 47 students sort 65 separate cards which had individual bits of knowledge from the schemata of

Ampere’s Law written on them. Students were asked to sort the cards into piles where cards in each pile were more strongly connected to one another than cards on other piles. The piles were then analyzed and compared to exam results finding strong correlations between success on the two tasks. A separate analysis was performed – “hierarchical cluster analysis” of the cards to independently determine the characteristics of the piles of cards for high performing students (70% and up) to low performing students (30% and below.) The piles created by the high performers agreed with the problem schemata in most cases and the piles for the lower students showed little agreement with the schemata. Finally they had students label each pile, and these results were consistent with Chi’s. The good students labeled using descriptions such as “related to induction” while the poor students used labels like “containing the word field”.

There have also been spin-off studies where researchers attempt to improve students’ categorization abilities in an attempt to improve problem solving ability (Bunce, Gabel and Samuel, 1991; Leonard, Dufresne and Mestre, 1996; Schoenfeld, 1985). Bunce et al and Leonard et al. successfully improved their students’ ability to categorize problems by deep structure; however, interestingly enough, neither saw an improvement in problem solving by any other measure. The students can now categorize based on deep structure but are not any better at arriving at the correct solution to a problem. Schoenfeld (1985) was able to improve students’ ability to categorize problems based on deep structure *and* improve their ability to solve problems. With Schoenfeld, instruction was not limited to categorizing problems but included modeling of various strategies/heuristics - how and when to use them - as well as stressing metacognitive processing.

This early literature includes papers that describe possible ways that knowledge could be stored based on the results of the card sorting studies. One hypothesis is that inexperienced solvers do not yet have complete conceptual understanding or possibly organize their knowledge around surface features while good solvers organize their knowledge around problem schemata (de Jong and Ferguson-Hessler, 1986). Ferguson-Hessler and de Jong (1987) define three types of discipline specific knowledge: *Declarative Knowledge* (facts), *procedural knowledge* and *problem situations* (concepts). They suggest that knowledge organization for successful students needs to be in the form of schemata that include all three types of knowledge listed above organized together so that various problems of the same type can be successfully tackled. The authors contend that later, for example in more advanced courses, students will learn enough about the subject to make the connections that create a knowledge hierarchy similar to that found in professional physicists. Larkin (1979) believes that experts knowledge is organized in chunks and thus when one piece is accessed, many others bits of relevant information is immediately available as evidenced by two experts using many equations at once and a student taking time between each new equation that they use.

These classic papers by Chi, Feltovich, & Glaser, deJong, Larkin etc… are often referenced. The articles themselves are careful to clarify conclusions, theories and opinions as to possible explanations while those who reference them commonly fail to differentiate between what is supported by the research and what is offered as one possible explanation. Gick (1986) is a good example. She states that Chi et al "have shown that whereas novices' schemata for physics problems are based on superficial similarity, experts' schemata are based on solution principles." This idea of differing schemata between experts and novices is most certainly not "shown" by this data. Further more, the authors of the cited works didn't even propose this idea.

These classic papers are based on small studies and include a lot of "thinking" about knowledge structure. They are valuable but only to the extent of providing ideas. It's such a tantalizing idea that something as straight forward as teaching problem categorization by deep structure could help students solve problems that many researchers latched on to this idea. This resulted in the idea gaining much more importance than was warranted. Hence the many studies which focus on improving students' ability to categorize problems. It seems pretty likely that many less experienced solvers are missing many skills in addition to their ability to categorize problems (if that is a skill). However, it is arguable that there could be times that the ability to appropriately identify the concept may be one of the only things missing. An example was discussed by Sing (2007) whose graduate students felt that help categorizing Jackson problems would have made all the difference in whether or not they could solve the problems.

There are many studies that use the results of Chi et al's card sorting but I have not seen any attempts in physics to offer alternative explanations to knowledge structure or additional reasons why less experienced solvers sort by surface feature or problem difficulty. Those who attempt to replicate the study and find that their students do better than observed by Chi et al. tend to explain the results as indicating that their students are stronger students than most. This may be true; however, I believe there are more likely explanations and some important factors one should consider here. The first that I'll mention is to look at the different directions that are given to the students. Chi et al's directions were to sort any way that made sense to the student. Less experienced students may believe it is more useful to sort by difficulty. Another possible reason is pointed out by Ross (2007). We commonly use the same surface features in problems when teaching a particular topic. Students are trained to associate Newton's Laws with inclined plane problems. It is perfectly reasonable for a student, with their experience having been limited to a typical physics course, to call this type of problem an inclined plane problem rather than a force problem. As an example, he was interviewing a math teacher (expert) and asked her to solve a problem where a person is moving up the "down escalator". As she solved it she referred to the person moving upstream because when she had learned about relative velocity (and when she teaches it) the problems involved rivers and boats. Chi et al's classic study provides some useful insight into the differences between less experienced and more experienced solvers but is only a very small piece of the puzzle.

Current Theories about knowledge structure

Current work includes some very nice theories about how we build a connected set of principles or a concept in our minds. Many theorists hold the idea that knowledge is a complex system of many kinds and pieces of knowledge elements and structures. Learning a concept requires coordinating a large number of elements in many ways. There is not a sharp line between having and not having a concept, in fact it's arguable that it's not possible to have a perfectly formed concept. A person can demonstrate reasonable competence without understanding every nuance of a concept. A small piece could be missing or a nuance malformed.

DiSessa and Wagner (2005) describe a coordination class as "a particular kind of concept whose principal function is to allow people to read a particular class of information out of situations in the world." Not all concepts are coordination classes. The concept of magnitude and direction of force or the expected value of a random variable are concepts that would qualify as coordination classes. The key is that these concepts are used to gather information about the situation. In general, physical and mathematical quantities are good candidates for coordination classes. Theorems, laws and other relational cognitive entities are usually not a coordination class because they are made of multiple kinds of information and relations. Some concepts only entail identifying examples of a particular concept which would not qualify as a coordination class. This is common in philosophical literature. Scientists certainly need to be able to do this with their concepts; however, this is not enough to demonstrate proficiency. For example, a student studied by diSessa and Sherin (1998) was able to consistently identify forces but was not able to accurately determine magnitude, direction or nature of forces.

Some may ask how a coordination class and a *schema* are different. Based on the definition in the literature of a schema - problem type and their associated moves (Ward & Sweller, 1990) - a schema includes factual and conceptual information as well as procedures. A coordination class is only a concept. There are also differences in the way that a schemata and coordination class are expected to be used due to differences in their defined structure. Current theorists such as diSessa also note weaknesses in theories that treat naïve concepts and professional concepts evenhandedly (i.e. those that treat conceptual change on the same level as shifts in theories.) This does not make sense when one considers a naïve concept and realizes that these are not well formed; however a theory clearly is.

When considering the classic theories of knowledge schemata such as experts' knowledge being in chunks, many researchers found it reasonable to believe that improving categorization skills could accelerate students' acquisition of expert like chunks of knowledge. Based on these theories, they also believed that an ability to categorize problems is an indicator of students' problem solving ability. However, the studies that improve their ability to categorize failed to show improvement in problem solving ability. If one considers diSessa's coordination class theory and his description of the knowledge structure of an expert, it is actually unreasonable to expect categorization to be a measure of a person's ability to solve problems. DiSessa exemplifies the idea between readout strategies and the causal net when he references Metz (1993) who shows that children understand that certain information is relevant before being able to make adequate use of it. "The distinction between inferences of existence and relevance on the one hand, and specific inference of information on the other, generalizes this developmental observation." (diSessa and Wagner, 2005) When translated this means: A person may recognize a conversation as being in French, but cannot understand it because they do not understand French.

There are other theories about how we organize knowledge (Hammer, 1996) and there is also an entire body of research on how to teach accurate ideas and the misconceptions students hold before instruction on specific topics in physics (Arons, 1981; Arons, 1990; Heller & Reif, 1984; Hewitt, 1990, 1994; Mazur 1996, 1997a, 1997b). Various lists and taxonomies of widely held ideas by students regardless of culture have also been compiled (Clement, 1982; DiSessa, 1981; Driver, Squires, Rushworth, and Wood-Robinson, 1994; Maloney, 1990; McCloskey, 1982). Having incomplete or inaccurate knowledge about a topic will clearly affect problem solving; however, I am focusing specifically on problem solving skills that are separate from content. Therefore discussion of these theoretical frameworks of knowledge and the many specific studies of topic specific knowledge acquisition are outside the scope of this literature review.

STRATEGIC KNOWLEDGE

Polya (1945) reintroduced the importance of heuristics in the field of mathematics and Schoenfeld (1985) did extensive work advancing our understanding of heuristics (as well as other important problem solving skills which will be discussed where applicable). Strategies or heuristics are only one piece of the puzzle but are easily the focus of over half of the problem solving literature in physics.

First I'd like to clarify the definition of a strategy/heuristic.

> "Heuristic, or heuretic, or "ars inveniendi" was the name of a certain branch of study, not very clearly circumscribed, belonging to logic, or to philosophy, or to psychology, often outlined, seldom presented in detail, and as good as forgotten today. The aim of heuristic is to study the methods and rules of discovery and invention… Heuristic, as an adjective, means "serving to discover." (Polya, 1945)

The typical physics text book includes a box with a problem solving strategy for each topic in every chapter. Since many of these are content specific (Serway and Faughn, 2000; Wilson and Buffa, 2003) they are actually not problem solving strategies as defined, they are procedures (see next section). Some include general strategies as well (Wilson and Buffa, 2003). Even though many of these are actually procedures, it is still indicative of the common belief that getting students to follow the right steps will make them good solvers. Larkin (1980) says "Perhaps the most intriguing idea in work on problem solving is that there might be very general and very powerful strategies underlying skillful problem solving in a wide variety of contexts." Effective strategies have also been studied in other non-scientific fields. Strategies such as breaking problems into smaller parts, designing a plan for composing an essay or techniques for memorizing a list of definitions have many features in common with problem solving strategies in physics (Mayer, 2003). These common skills can be seen when narrowing your focus.

Early problem solving research in physics and math is summed up by Larkin (1980) where she considers three examples of general strategies. First, means-ends analysis (defined below) has been demonstrated to be an effective strategy when solving an unfamiliar problem (Simon and Simon, 1978). Second is a type of planning where the original problem is replaced with an abstracted version in which certain central features are retained and then this solution is used to guide the solution to the original problem. This strategy has been modeled by Larkin et al. (Larkin, 1977; McDermott and Larkin, 1978) and shown to be an effective means for a computer to solve many different kinds of problems; Third the use of goals and subgoals. Greeno (1976) has developed a successful model for solving geometry problems using subgoals. Larkin's conclusions are believable, but unfortunately the studies that are cited have less than three subjects per study and are usually focused on computer models. More work needs to be done to substantiate these claims.

Is there One effective problem solving strategy?

Many researchers have implemented a (single) problem strategy in their classrooms (Bunce and Heikkinen, 1986; Heller, Keith, Anderson, 1992; Heller and Hollabaugh, 1992; Heller and Reif, 1984; DiLisi, Eulberg, Lanese and Padovan, 2006; Huffman, 1997; Leonard, Dufresne and Mestre, 1996). These studies have met with varied success seeing improvement in the use of the prescribed strategy; but, this does not seem to translate into success at solving the problems. A typical strategy is as follows:

1. Visualize the problem
2. Create a physics description - draw a picture
3. Plan a solution – choose appropriate algorithm
4. Execute the plan
5. Check and evaluate

After explicit instruction on the strategy, students use written representations more often and students' ability to choose the appropriate concept/principle is improved; however, improvement at identifying the principle did not help students solve the problem. In all cases, students expressed annoyance and frustration with having to use the strategy. They stated that it was cumbersome, time consuming and did not seem to help. Related studies, also using a prescribed strategy but with more difficult problems, found that students are less resistant to the strategy. These students say that it was more useful for harder text-book type problems and context-rich problems (Heller and Hollabaugh, 1992).

Are there a variety of effective problem solving strategies?

In contrast to a single strategy, Schoenfeld (1985) teaches a variety of heuristics and how to recognize which would be most effective, depending on the problem situation. Schoenfeld has done extensive and quite detailed studies of problem solving; however, his study size is limited. Most studies cited typically involve between 7 and 20 students. In one study, conducted at UC Berkley, Schoenfeld gave four students very specific training on five useful heuristics and provided continued instruction on the usefulness of these heuristics as they solved the problems. As a control, three other students solved the same problems but only received training on the heuristics at the beginning. Later testing showed that the students whose training focused on the five strategies did much better. One of Schoenfeld's claims from this study is that regular practice is not adequate for learning. I don't see that that particular claim to be supported by this study or others. The students who focused on the heuristics, received the four problems that applied to one heuristic at one time. The control group received all the problems in mixed order so that the effective strategy was not seen twice in a row (20 problems with five strategies mixed together). This study does demonstrate that the students learned how to use most of the heuristic strategies in the experimental group; however, the study does not verify the claim that the students don't make use of the strategies without the specific training due to the mixed ordering of problems in one case and not the other.

A more recent study by Tuminaro and Redish (2007) focuses on what they term as epistemic-games (e-games). These e-games are the strategies that groups use while solving problems. The e-games were identified through video analysis of introductory students solving problems. The problems were well-defined but set in a real-life scenario and were designed to be too difficult for a student to solve on their own. The e-games that were identified include both productive and un-productive strategies such as "mapping mathematics to meaning" or "recursive plug-and-chug." The goal is to help teachers identify and enhance productive games; however, it requires extensive review of the videos to identify which game the students are engaged in and these games are identified for the group as a unit. Depending on which student is being watched at the time or the direction the discussion turns, the game varies. Hence, this work in its current stage has limited value in evaluating or improving student problem solving.

There is a prevalent belief that teaching the appropriate strategy to students will make them into good problem solvers. This seems to be a reasonable idea on the surface since poor solvers do not engage in what appear to be useful strategies while solving problems. It is only natural to hope for a simple cure of teaching them to behave as successful solvers. If it were that easy, we wouldn't still be struggling with this issue. Schwartz, Bransford and Sears (2005) point out the complexities, saying,

> "There are a host of useful and sophisticated problem-solving routines. Nevertheless, they still are often taught as script-like, mechanical routines-often because this is the only way to show effects when they are assessed through the efficiency-oriented lenses of applicative problem solving…. [Students need to] learn to break free of old routines and discover new ideas on their own."

It is clear from the above studies that simply showing students a strategy for how to solve a problem doesn't make them better at finding the answer to a new problem. There is a deeper question of causality. Why are they not using the same strategy as a successful solver? As described earlier problem solving requires many seperate component skills. A weak solver could be missing one or more of these skills. If they don't have strengths in all the necessary skills for a particular problem, they must improvise and try to find a way to use whatever abilities they may have. When discussing readout strategies (how students determine the useful information), diSessa suggests that quite often "try and see" is the most appropriate heuristic when learning which naïve knowledge elements (p-prims) will be useful for producing a well-formed coordination class. So making novice problem solvers behave as if they know how to solve a problem or even treating it as an exercise as it would be for an expert, is not the right solution. It doesn't fix the missing skills. A crude but applicable analogy would be to consider an actor who plays a doctor in a movie. He can observe doctors and become quite believable in his part; but, this does not mean he is capable of doing anything to help a heart attack victim, simply because he was trained to behave like a doctor.

Studies of means-ends analysis and expert novice differences

Although there are many heuristic strategies available for solving problems, means-ends analysis warrants its own section because there is a large body of research on this particular strategy in physics. It is common to see the claim that means-ends analysis is a weak strategy only employed by novice/poor problem solvers (Maloney, 1993; Gick, 1986) The definition of means-ends analysis is

> "…means-ends analysis involves assessing the difference between the current state of knowledge about the problem and the state of knowledge required for the problem's solution. This assessment is then used to select an action that reduces any difference between these two states of knowledge." (Larkin, 1980).

In physics, early research on the comparison of more experienced problem solvers (experts) and less experienced (novices) included the observation that less experienced solvers use means-ends analysis while experienced solvers work forward. This observation has been cited extensively and become part of the common lore on "experts versus novices" Since physicists hold this belief as well as the fact that typical back of the chapter text-book problems encourage this strategy, it has motivated a disproportionately large amount of work on this topic.

There are a handful of studies that are commonly referred to in support of the above statement that means-end analysis is a weak strategy (Simon & Simon 1978; Larkin & Reif, 1979; Larkin, 1980). However, often it has not been properly appreciated what this work did and did not show. First, all of these studies are limited to less than five subjects and the definition of the experienced subjects and novice subjects varies and in some cases is not even defined. Second, the results of the studies are often misinterpreted. For example, a quote taken from Larkin, McDermott, Simon and Simon to support the claim that means-ends analysis is a weak technique used by problem solving novices is:

> "…working backward is usually thought to be a more sophisticated strategy than working forward. But experts work forward…"

A closer look at the classic research in physics that is often referenced when making the claim that strong solvers always work forward, reveals that the literature is sometimes not referenced accurately. As an example, the above quote in its entirety reads:

> "A second difference, verified from their worksheets and the thinking-aloud protocols they produced, was that the novice solved most of the problems by working backward from the unknown problem solution to the given quantities, while the expert usually worked forward from the givens to the desired quantities. This was surprising, since working back-ward is usually thought to be a more sophisticated strategy than working forward. But experts work forward only on easy problems, where experience assures them that, without any particular planning, solving all possible equations will lead them quickly to a full understanding of the situation, including finding the particular quantity they are asked to solve for. They thus solve the problem by accumulating knowledge about the quantities that were initially unknown. Novices, having little experience with kinematics, seem to require goals and subgoals to direct their search. The management of goals and subgoals – deciding periodically what to do next – may occupy considerable time and place a substantial burden on limited short-term memory." (Larkin, 1979)

Clearly Larkin does not consider means-end analysis to be weak, rather she says "more sophisticated". Further discussion in this paper hypothesizes that the expert's knowledge is chunked so that when one piece is accessed, the rest of the information on this concept is immediately available. In later research Larkin (1980) finds that the more difficult a problem is for an expert, the more likely they are to use means-ends analysis.

The strongest opponent to means-ends analysis is probably Sweller who specifically states that "means-ends analysis retards learning due to cognitive load" (Sweller, 1988; Sweller and Levine, 1982; Ward and Sweller, 1990). When Sweller studied the effects of knowing the goal versus hiding the goal from the solver, he used a variety of problems ranging from finger mazes, computer mazes to number patterns where the goal has very little to do with the correct solution procedure. These problems could be considered misleading, if not outright trick, problems. Sweller uses these studies to support his hypothesis that means-ends analysis blocks learning by creating additional cognitive load. His studies were very carefully performed; however, the problems he chooses are not relevant to solving typical physics problems. The goal and the required solution pattern in Sweller's problems either have no relation or in many cases, such as the maze problems, the goal is clear mis-direction from the solution pattern. Means-ends analysis involves working backward from the goal state to find a solution. Therefore it is not possible to use means-ends analysis on the problems that Sweller creates for these studies of the effectiveness of means-ends analysis. Sweller's ideas about the importance of understanding a problem's structure and the solver's cognitive load are very interesting ideas; however, due to the nature of his studies, is not supported by his empirical results. His studies however, do bring up many other interesting points about problem solving. See discussion of beliefs below for additional reasons why these types of problems are particularly difficult to solve and learn from.

In contrast to Sweller, Newell and Simon (1972) and Schoenfeld (1985) as well as many others consider means-ends analysis an effective heuristic for students to employ when attempting to solve problems. Schoenfeld taught students how to use this strategy and was able to show improved problem solving ability when this method was used. Pretz, Naples and Sternberg (2003) discuss that when working backward on the Tower of Hanoi problem[1] one can quickly discover the necessary

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[1] There are three discs of unequal sizes, positioned on the leftmost of three pegs, such that the largest disc is at the bottom, the middle-sized disc is in the middle, and the smallest disc is on the top. Your

procedure for solution. Using means-ends analysis, one realizes the solution is very straight forward.

Mayer (2003) hypothesizes that experienced problem solvers work forward because they are familiar with the particular problem area and store their factual knowledge in large units, consistent with Larkin. If this is the case, experienced solvers can simply add specific values to the well formed, large units of knowledge they have and essentially automatically solve the problem. This idea is supported by the results of Chi, Feltovich and Glaser's (1981) classic problem categorization research discussed above. Mayer further discusses the possibility that when working an unfamiliar problem, the necessary knowledge is in small units so the solver "must figure out how to put the equations together, a process that lends itself to working form the unknown back to the goal." This is supported by both Larkin's findings of experts using means-end analysis as problems get more difficult as well as research in medical fields that reveals that someone who's considered an expert will still work backward when they are given an unfamiliar problem (Groen & Patel, 1988).

This is not to say the means-ends analysis is the best strategy when faced with a real problem. Schoenfeld (1985) cautions that this strategy is only effective when 1) working within the context of a clearly defined search space, 2) the solver is clearly aware of the tools at their disposal, and 3) they have a clearly defined goal structure. Sweller's "trick-problems" definitely don't fall within this definition so would not be expected to be successfully solved via means-end analysis. However,

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task is to transfer all three discs to the rightmost peg, using the middle peg as a stationing area, as needed. You may move only one disc at a time, and you may never move a larger disc on top of a smaller disc. (Sternberg, 1999) Can be expanded to use many pegs or discs of varying sizes but still requires the same solution procedure.

most back of the chapter physics problems as well as the typical problem presented to school children (Pretz, Naples & Sternberg, 2003) fall within this category. So it is not surprising that students often use means-end analysis in our physics classes. For those problems in that context, that is likely to be the most efficient, successful approach.

PROCEDURAL

Procedural knowledge has slightly different, although complimentary, definitions in physics and cognitive science. Physics education researchers define procedures as the allowed moves when solving a problem. Specifically, Larkin (1979) says procedures are condition-action units. Whenever a current situation satisfies a specific condition, the corresponding action is implemented. Behavioral studies and neuroscience studies show that procedural and declarative knowledge are specialized systems that have different purposes and are located in different regions of the brain. Procedural memories or productions (Anderson & Schunn; 2000) are the knowing how and come about only from extended practice.

In Physics

Many physics educators consider procedural knowledge important and have tried to teach procedural knowledge by telling. Text books contain problem solving boxes with procedures listed for each topic (Korinsky, 2003). Ferguson-Hessler and deJong (1987) describe people's organization of knowledge structure as schemata or hierarchical. They contend that explicit use of problem solving schemata will help teachers make their tacit knowledge (implicit knowledge – bits and pieces we don't have names for but we ‘know’) visible to the students and, consequently, will enable students to realize that characteristics of problem situations are used to determine which principles to apply.

On the other hand, much of the more recent physics literature on problem solving discusses students’ use of procedures or algorithms for solving specific types of problems as something that is not desired (Heller, Keith, Anderson, 1992; Heller and Hollabaugh, 1992; Cohen et al, 2000; Frank, Baker and Herron; 1987.) These researchers observe students who use algorithms to solve problems and see that students don’t seem to believe they need to think about the reasonableness of their solution etc…. (See section on beliefs for related discussion.) These researchers have hypothesized that teaching a problem solving strategy can help fix this ‘problem’ of students believing that learning algorithmic procedures is doing physics.

Cognitive Psychologists

Schwartz, Bransford and Sears (2003) believe that fault lies with the problems we typically offer rather than with the students. If we design problems that can be solved with algorithms students learn to use them. Algorithms are useful because they are one skill that’s needed - efficiency. Our current form of evaluation is only capable of measuring these sorts of skills. However, if one finds other methods of measurement that value other general skills then educators will be encouraged to develop problems that will help students learn these skills. (See section on processing and Chapter IV for more detail on Schwartz et. al.)

Cognitive psychologists have studied and carefully delineated the differences between declarative and procedural knowledge. Declarative knowledge is relational, it's quick to learn, can be accessed flexibly and can be used flexibly. In contrast procedurdal knowledge is slow to learn, requires specific circumstances to access and has specific uses (not flexible). For example, a physics graduate student can manipulate algebraic equations quickly with little thought; however, to explain the rules s/he's using to another would be very difficult. Most likely the student would have to go through the manipulation and see what they are doing to explain in detail. Clear examples demonstrating that these two systems of memory are distinct come from the study of amnesiacs who are unable to form declarative memories (Cohen and Eichenbaum, 1993). For example, one study describes an individual who was given the task of tracing around the lines of a star while looking at the reflection of his hand in a mirror. Each day the subject came into the task and expressed that this was the first time he'd ever had to do anything like this; however, each day he made fewer errors. The first attempt had 30 errors and by the third day he averaged less than one error each time he performed the task.

Anderson and Schunn (2000) describe the ACT-R theory that specifically calls procedural knowledge *production rules* and describes how these rules are acquired and then retrieved. A summary of the description of a production rule would be that production rules are created through *analogy*. This means a goal is required and an example of the solution. Just giving an example does not guarantee that a person can create a production rule. They need to understand the example and to deploy it. They need to see that the example applies to the new situation. The authors go further to discuss that storing information once is not enough. It must also be used many times - up to 40 times for the same task over and over.

Mayer and Wittrock (2006) say automaticity methods are useful such as memorizing times tables. This frees up working memory for problem solving tasks such as devising and monitoring a solution plan. Researchers have taught students to automate decoding of passages while reading by having them read out-loud over and over – consequently the students were better at comprehending passages after automating these procedures (Samuels, 1979; LaBerge & Samuels, 1974). Similar work in math showed automating component skills, such as recognizing congruent angles, allows students to progress from effortful performance to automatic performance. This frees up capacity to focus on the problem solving (Singley & Anderson, 1989). The key is to provide problems that encourage automaticity, when you are having the students learn new procedures, as well as more difficult problems, that use these algorithms but also *require* more skills to find a solution. Ericsson (2003, 2007) describes how an elite expert has undergone 10,000 hours of *deliberate practice* - repetition and extension of one's current skills. This is in part due to the many procedures that must become automated before a person is expert at something. These can be physical procedures (performing a triple axle) or mental such as taking a simply derivative.

Using the lens of procedural vs. declarative memory

Applying the information that has been gathered about the differences between procedural and declarative memories, where they are located and how they are created and accessed, to the different research in physics provides further understanding of these studies. These studies each include ideas about how knowledge is structured and ways to improve teaching. Researchers have made many attempts to improve students' ability to solve problems based on these ideas; but, have found limited success. The information on procedural and declarative memory provide more detail about knowledge structure and reasoning that elucidates this lack of success.

Using the ideas of procedural versus declarative memories as defined by cognitive psychologists, provides an alternative explanation to Larkin et al.'s (1979) chunking. Larking et al. found that one of the differences between experts and a novice was that when the experts used equations, they used several different ones without hesitation while the novice would decide on one, think about things (use it or not) and then decide on the next one. Larkin hypothesized that the fluid use of equations was an indication of the connections that existed between the bits of information or knowledge structure. Another idea, related to how the information is stored, is that maybe these equations had become routine and was now part of the experts' procedural memory. The novice, on the other hand, lacked enough practice and had to depend predominantly on their declarative knowledge to solve the problems. It is hard to learn about expert problem solving skills if the different groups of subjects are performing different tasks – exercises versus problems.

Ferguson-Hessler and deJong say making 'tacit' knowledge visible is useful; however, it seems in view of the above description of procedural knowledge making the tacit visible is not a simple and complete solution. Rather it's also practice of these specific procedures that is needed. In addition, when this knowledge is procedural for the instructor, it's actually quite difficult for the teacher to recognize every step of what they are doing when performing a procedure so that they can make it visible.

Ward and Sweller (1990) demonstrated that students do not automatically pick up on problem structure (the effective moves and rules - procedure) just by solving typical problems. They demonstrated that worked examples help but that the specific type of worked example was critical. The worked examples that they demonstrated as effective had the solution integrated with the problem statement, thereby providing clear connections between the variables, equations and problem statement. The authors' contend that schema acquisition could occur in these cases due to reduced cognitive load. From an examination of the good versus bad worked examples given in this paper, it appears that the good examples helped provide additional facts, bits of information, to the students thus facilitating transfer of the knowledge to future problems. As Anderson and Schunn describe, to create a procedural memory students need to understand the example and *then* practice it. Cognitive load is important but does not appear to be the barrier in this case.

Beliefs, Expectations and Motivation

Beliefs and expectations are an important area of research with a substantial impact on solving problems. However, it is often overlooked by educators. diSessa and Wagner (2005) observed that researchers seldom if ever ask subjects for confidence ratings. In a perfect world knowing the applicable facts and concepts along with procedures and strategies should solve most problems, but we know from experience that this is not the case. A person brings with them ideas about their own abilities, about the environment (test vs. homework), about the discipline ("Math problems should take less than 5 minutes." or "You just plug in the values and get an answer.") etc… that they've learned from previous experiences including socio-cultural influences. This "world view" affects how the solver engages or even *if* they engage in solving the given problem. It includes their view of themselves, the problem and the nature of science (or whatever discipline the problem is framed within), which will determine their behavior. Schoenfeld (1985) expressed this very clearly,

> "Purely cognitive behavior – the kind of intellectual performance characterized by discussion of resources, heuristics, and control alone – is rare. The performance of most intellectual tasks takes place within the context established by one's perspective regarding the nature of those tasks. Belief systems shape cognition, even when one is not consciously aware of holding those beliefs."

Schoenfeld divided beliefs into four categories, which include: Beliefs about *self, the environment, the topic,* and *mathematics [the discipline]*.

I prefer breaking beliefs into two categories - about self and about the problem. There are many situations where I have a hard time choosing only one of Schoenfeld's four categories. For instance performance goals, as defined by Dweck (1999), are when a student behaves differently depending on the different expectations of being evaluated versus a learning situation. This could fit under self or environment but, using two categories only, it easily goes under self. Another example would be perceived difficulty level of a problem which could be caused by the environment, the topic or the discipline. However, the category of problem would be appropriate in all cases. So I find the two general divisions of self and the problem more productive for framing the research. Admittedly, there are many specific categories within each.

BELIEFS ABOUT SELF

Metacognitive Knowledge

There are many aspects to ones beliefs about self – knowledge of owns self as a problem solver. These include procedures you are good at, strategies you can rely on, confidence, and intelligence (fixed versus malleable). One could argue that the ideas of beliefs about oneself have as strong an impact on problem solving as knowledge, in that weaknesses in either is usually fatal. With resources (facts, concepts, procedures) if you don't know them, you can't get an answer. Beliefs can be just as debilitating. If the solver doesn't think they can solve the problem or is just plain not interested, then they won't.

When Schoenfeld (1987) defined metacognition, he said it had three related but distinct categories of intellectual behavior: 1. Your knowledge about your own thought processes. How accurate are you in describing your own thinking? 2. Control, or self-regulation. How well do you keep track of what you're doing when (for example) you're solving problems, and how well (if at all) do you use the input from those observations to guide your problem solving actions? 3. Beliefs and intuitions. What ideas about mathematics do you bring to your work in mathematics, and how does that shape the way that you do mathematics? These definitions are somewhat circular since beliefs are within metacognition here but in his previous work (Schoenfeld, 1985) beliefs included item 3 above as well as beliefs about self – arguably item 1 above. In Schoenfeld (1987) he describes metacognition as the umbrella category that subsumes much of the other categories used to describe problem-solving performance. I find such a broad definition somewhat awkward to use as well as a bit inconsistent with some of the literature. The portion of his definition listed as Item 1 above is what I'm defining as metacognitive knowledge and seems to be defined consistently in the literature. Item 2 is where the literature does not always agree and where a more precise definition can highlight a sometimes overlooked but quite valuable processing skill that I've discussed below called meta-processing. Item 3 is the portion of beliefs that I've labeled beliefs about the problem that is discussed below.

The definition that beliefs about self are metacognitive skills is consistent in the literature both in science and psychology. This includes an understanding of ones' strengths and weaknesses when solving problems. Brown, Bransford, Ferrara and Campione (1983) say that "Knowledge about cognition refers to the relatively stable, statable, often fallible, and late-developing information that human thinkers have about their own cognitive processes and those of others." Dweck (1999) frames beliefs as "…the idea that people develop beliefs that organize their world and give meaning to their experiences. These beliefs may be called "meaning systems", and different people create different meaning systems (their *self-theories*)."

Berardi-Coletta et al. carefully studied metacognitive skills and metacognitive processing. They also describe the difference between the skill and the thinking process before showing in detail how the processes help facilitate problem solving

(Berardi-Coletta, Buyer, Dominowski and Rellinger, 1995). Other researchers such as Mayer and Wittrock also carefully describe the differences. Metacognitive knowledge is a declarative type of knowledge, something the student knows about themselves (may be accurate or not) and is very distinct from thinking about one's processing which is described in detail in the cognitive processing section.

Expectations – Beliefs in action

There is a lot involved in expectations with some complicated scenarios that have unique results; but, all boil down to students engaging in unproductive behaviors because of various expectations. Elliot and Dweck (1988) gave a group of 5^{th} graders the same task. Half were told they would be evaluated on how they did (performance goal) and the others were told they would learn useful things from the task (learning goal). Many students with the performance goal showed a clear helpless pattern in response to difficulty. Some condemned their ability and their problem solving deteriorated. Most of the learning goal students showed a clear mastery-oriented pattern. They did not worry in the face of failure and remained focused on the task.

Next they took half the performance goal students and told them they had high ability in this area and told half that for now they had low ability in this area. They did the same with the learning goal group. For students with performance goals, those who were certain of their high ability held on in the face of difficulty. The students who thought their ability was lower fell into a helpless response. For students with learning goals the message made no difference.

In a previous study Farrell and Dweck (1985) initially evaluated 5th grade students on their persistence in the face of difficulty. Based on this evaluation she divided the students into two groups *helpless* versus *mastery* response. The researchers then gave both groups 12 problems, the first eight were at the appropriate level for these students and the next four were very challenging. The helpless group did equivalent on the first eight. On the four challenging questions the helpless students gave up, said disparaging things about themselves and changed the subject to things that made them look good “I’m going to be an heiress.” After the students went in to the helpless mode, the interviewer asked these students if they could go back to the problems in the beginning (the eight they’d already solved) and solve them again. Most could not. The mastery group could not solve the four challenge questions but kept trying. After failing at these four, the interviewer asked these students if they could go back and solve the first eight and they all said of course, many considered this a ridiculous question.

Adams et al. (2008) finds that students have a similar unproductive response while interacting with computer simulations that they have previously seen and believe they understand. These students were certain they “knew” the material but it had been several months since they’d interacted with the simulation so were rusty on the ideas. In previous interviews these students were confident capable students. When interacting with the same simulation several months later, the students fell into a *performance mode*, attempting to recall ideas, rather than *engaged exploration* - learning mode; the mode of behavior that they had previously displayed during several other simulation interivews. When in performance mode students did not explore the simulation or attempt to make connections. They were trying to retrieve information from memory and wouldn't allow themselves to process. When the students could not immediately recall the ideas, they continued to look quickly and unproductively through the simulation while they came up with reasons/excuses for their inability to recall. These reasons ranged from, they didn't like this simulation, to they were sick for that lecture. These students never attacked themselves or their abilities (as demonstrated by Dweck's helpless behavior).

Dweck's performance goals and Adams et al.'s performance mode are similar but subtly different. Both describe situations where expectations can create complete failure to solve a problem when all other skills are available to that student. Adams et al report on students expectations about their knowledge level of the content while Dweck studies students' reactions when the task determines the expectations of students general approach - helpless vs. mastery. The reactions of the students differ slightly in each case but in the end, are unproductive entirely because of the students' expectations. Dweck's students act helpless in the face of failure. Dweck sees students quit everything or change the rules, "I'm picking brown because I like chocolate cake.", and are unable to do tasks they were previously capable of doing. Adams et al's students don't give up but create reasons why their failure is out of their control. Because of their expectations about knowing, they get agitated and won't process.

In an attempt to further understand students' response to challenging situations, Sorich and Dweck (1999) studied students in their first year of junior high school. Students were given a questionnaire that was used to classify them as either “entity theorists” or “incremental theorists.” The authors define entity theorists as those who believe we are born with a set amount of intelligence while incremental theorists believe that learning requires work. The two groups’ confidence and achievement tests in English and Math were compared. Both groups were equivalent on all measures. At the end of the year the students’ grades were compared and the entity theorists received almost a full grade lower in both math and English. Having equivalent confidence was not enough. Dweck et al. has found that entity theorists may do well when things are not too difficult but once they reach a new level of difficulty, they give up or change their focus to an easier goal.

Other work on expectations shows similar effects from *stereotype threat*. Steele, Aronson and Spencer (Aronson, Quinn & Spencer, 1998; Steele & Aronson, 1995; Steele, 1997) proposed the idea of stereotype threat and how it can shape intellectual identities. Steele (Steele, 1997) administered a test to math majors that had problems slightly above their abilities. With one group he told the students this is “just a trial” and we want to “see how you do”. With the second group he told them that the test “showed gender differences”. The no threat group had equivalent performance for both men and women with an average score of 17 questions correct. The gender threat group had an average score of 5 for the women and 27 for the men.

Aronson and Fried (1998) studied stereotype threat at Stanford. They reason that the transition from high school to college, especially a competitive college such as Stanford, can be a challenging one for African American students. For their study they showed a short film to both African American and Caucasian students presenting scientific explanations, researchers’ testimonies, neurological graphics and research findings to the effect that every time people meet a challenge, exert mental effort, and learn something new, their brain grows neurons and they become smarter. The film was accompanied by a lecture and the students were required to write a letter to grade school students explaining their new view of intelligence and how it expands with work. At the end of the term Aronson and Fried compared the grades earned by all students who had seen the incremental film to the grades of students who had not. Of the students who had not seen the film, the GPA's of the Caucasian students was significantly higher than those of the African American students. For the students who had seen the film, the gap between the majority and minority students was appreciably reduced. In addition, the black students reported enjoying school more and considered themselves more academically oriented than their peers in the control group.

Beliefs about self are far from thoroughly understood. There are many cues that can cause a student to give up or slip into unproductive behavior when a different belief was all that is needed for the student to engage in productive behavior.

BELIEFS ABOUT THE PROBLEM

Hard vs. Easy

Expectations make all the difference. Not only in how people approach problems, i.e. find it useful to understand the meaning of the equations, but also the problem's difficulty level. Studies have shown that if a student thinks a problem is easy or hard when it's the opposite, they will do worse than if they have the appropriate expectation. Reusser (1988) found that when students were told a hard problem was hard, they did much better than the students who were told the same problem was easy. Students who had the false impression, that it was easy, treated the problem superficially. Reusser also found that students used the information about whether their answers were “nice” numbers as queues to determine if they’d successfully solved the problem. This may not be what the teacher would like to see but this is problem solving. Students are using all the information they have at their disposal, including expectations about easy and hard problems.

Expectations About Problem Types

A study of junior high school students (Carpenter, Lindquist, Mathews and Sliver, 1983) exemplifies how student expectations can affect how they solve problems. 45,000 13 year olds were asked to solve the problem:

> An army bus holds 36 soldiers. If 1128 soldiers are being bused to their training site, how many buses are needed?

70% of the students did the division correctly; however, only 23% gave the correct answer of 32. 29% of the students gave the answer 31 remainder 12 while 31 was given as an answer by another 18%. It appears that these students see mathematical manipulation as what is important and what is being tested. These students are used to problems that require exact answers and either haven’t considered the reality of a third of a bus or believe this is the sort of answer the teacher really wants.

Pretz, Naples and Sternberg (2003) consider the following problem:

> You have a pitcher full of lemonade and a pitcher full of iced tea. You simultaneously empty both jugs into one large vat, yet the lemonade remains separate from the iced tea. How could this happen?

This problem is used to exemplify that fact that people make assumptions when solving problems. Most people will assume the lemonade and ice tea are both liquid when emptied into the vat, but the problem does not state this. If one considers the possibility that both are frozen, the problem is simple.

Schoenfeld (1985) had more specific examples of how expectations shaped problem solving. For example, he saw that students would only use empirical methods to test their hypotheses. Each time they came up with a new idea they'd construct a diagram to test it. When asked later why they didn't try deductive reasoning, the students said they "had not thought it could be useful." After the suggestion, the students were able to productively use this strategy.

A related but somewhat different concern with expectations is the importance of considering student expectations when designing studies of student behavior. Sweller has several papers on cognitive load, means-ends analysis and worked examples (discussed in strategy section above). In these papers he performs very careful experiments to demonstrate the hypothesis that means-ends analysis blocks learning. The first shortcoming of these studies is, due to the nature of the problems, it is not possible to solve these problems with means-ends analysis (discussed in the strategy section above). In addition, there are several important issues that arise with student expectations due to the nature of these problems. In the first set of studies students solve finger mazes and mazes on a computer. One group was shown the goal and the other group was not. The students used the goal to direct their solution; however, the path to the goal led away from the goal until the very end. Knowing where the goal was provided mis-direction and inevitably added to the required number of moves; but students did solve the problem. When both groups, goal and no-goal, were asked to resolve the problem, no difference was seen between the success of the two groups. Each group learned the pattern and the goal group also had to rule out the usefulness of knowing the goal. There are two sets of expectations here that muddy these studies: 1. The goal provides a clue as to which direction the students should head – but is actually the opposite direction of where they need to go until the very end; and 2. Mazes do not typically have patterns as part of their solution.

In summary, the literature clearly shows that students' expectations and assumptions have a strong influence on the type of solution they provide, strategies they attempt to use and even the way they read the problem. All of this is important to consider for understanding what students are thinking when they solve problems and when deciding how to teach.

Expectations about Physics Problems in Introductory Courses

Students come into physics courses with ideas about what will be necessary to learn physics. These ideas shape the way they learn. The importance of students' beliefs about learning physics has come to the forefront only recently and researchers are just beginning to create curricula that address these issues.

Hammer (1989) says, "If some students conceive of physics as a collection of isolated facts and formulas, it may never occur to them to pay attention to underlying reasoning." When writing this article, Hammer felt that the real problem in physics is the way courses are traditionally taught. He studied two female students who are both taking a traditional physics course. One of them finds it important to understand why and where things are coming from while the other is simply interested in learning how to solve the problems and doesn't look for connections or understanding. The first student does quite well during the first section of the course but has to work hard to find the meaning and connections and is often frustrated. Eventually this method hurts her grade in the course and she gives up on understanding and adopts the other student's method of learning the formulas and trusting whatever the professor offers as fact. The second student does consistently well because she does only what is necessary and takes what the professor offers at face value. She did not, however, come away feeling that physics was enjoyable or useful.

Hammer (1996) discusses the variables that affect student expectations. Independence - takes responsibility for constructing understanding vs. takes what is given by authorities without evaluation. Coherence - believes physics needs to be considered as a connected consistent framework vs. believes physics can be treated as unrelated facts or independent pieces. Concepts - stresses understanding of the underlying ideas and concepts vs. focuses on memorizing and using formulas without interpretation or sense making.

Leonard, Dufresne and Mestre (1996) believe part of the reason students do not gain a solid understanding of major ideas in physics lies in the way we model problems.

> "When modeling problem solving for students, although we are usually careful to state verbally the principle or concept being applied to solve a problem, we often only *write down* the equations by which the principle is instantiated. Students, therefore observe that it is the manipulation of equations that leads to solutions; their perception is that principles are abstractions that bear little relevance to obtaining answers to problems."

Additionally, students will continue to avoid the difficult task of attempting to understand the deep meaning of concepts in favor of more pragmatic goals, such as becoming proficient at manipulating equations to obtain answers to problems.

Schoenfeld (1985) was able to show that when students were explicitly shown effective strategies and how to recognize useful strategies, they used them. This demonstrates that these students either initially didn't realize that a particular strategy may help them or didn't know how to determine which will work. It does not support the commonly held idea that students don't use various strategies only because they choose not to spend the time.

MOTIVATION

Motivation is closely related to beliefs about self and the problem. Many times these beliefs and expectations are what create a student's motivation. The two are difficult to parse. When discussing problem solving, only a few researchers discuss motivation. Those who do divide things in different ways but recognize many of the same aspects and agree that the ideas that students bring with them about themselves and the problems they must solve have an immense impact on how they solve problems and learn.

In Mayer's early work (1998), he classifies problem solving into three main categories - *skill, metaskill* and *will.* This is in contrast to his more recent categorization of *knowledge* and *processes* (Mayer and Witrock, 2006). As mentioned earlier the division of knowledge and processes is what I'm using here as the foundation of my categorization; however, I've pulled beliefs and expectations out and put it into its own category. I then added motivation to the beliefs and expectations category. This division actually has many similarities to Mayer's previous choice of using skill, metaskill and will. The division between skill and metaskill was poorly defined; but, I felt will was clearly defined and quite valuable. I'm surprised that it is not part of his current categorization and that motivation is not mentioned in his 2006 review.

Skill included specific content knowledge, facts and procedures, as well as information processing, which included the cognitive tasks of encoding, inferring, applying and responding. The next area of *metaskill* included knowing how to ask the right questions and self monitoring of one's skills. Students may have all the necessary skills to execute the solution but do not know how to represent the problem and devise a solution plan. The challenge is knowing when and how to apply their factual and procedural knowledge. These two categories are difficult to parse so it makes sense that his current (Mayer & Whitrock, 2006) organization uses a different division, that of what a student has and what they do.

The idea of *will* is possibly the most complicated aspect of problem solving. There are many facets to *will* and Mayer groups them into three areas - interest, self-efficacy and attributions. A student could learn physics with motivation based on interest (internal motivation) or effort (external motivation). By teaching in terms of a student's personal experiences, one could hope to produce motivation based on interest. Self-efficacy refers to a person's judgment of his or her capabilities to solve problems. Finally, attributions, a word that could easily be replaced by the term excuses, refers to what the student sees as the cause of his failure - personal versus external responsibility. Typically a student who blames academic failure on a lack of effort is willing to work harder than one who blames failure on influences outside of their control.

Redish (2003) classifies the "hidden curriculum" as including expectations, metacognition and affect. Within expectations is a student's feelings about science and how they interpret what they hear. Metacognition is thinking about their own thinking (*reflection)* consciously – "these two sentences don't make sense" and subconsciously via confidence "it just feels right". Affect includes emotional responses including motivation, self-image and emotion. When defining affect, affect includes: *internally motivated* – self-motivated by an interest in the subject and a desire for learning; *externally motivated* – motivated to look smart (high score)*;* *weakly motivated* – doing something because it's a requirement, but only are concerned with getting by; and *negatively motivated* – want to fail. Maybe to demonstrate to a controlling parent or mentor that they are not suited to the subject.

Cognitive Processing

Knowledge or Cognitive processing? Which is more important? Korinsky (2003) concludes "…rigid knowledge and bisociation [cognitive processes] being two distinct sets of skills essential to problem solving: the notion of bisociation as the main factor limiting one's problem-solving success was also supported." This is one side. Greeno (1980) states the other side of the coin, "A person may not have learned exactly what to do in a specific problem situation, but whatever the person is able to do requires some knowledge, even if that knowledge may be in the form of general strategies for analyzing situations and attempting solutions."

Goldstein and Papert (1977) claim,

> "Today there has been a shift in paradigm. The fundamental problem of understanding intelligences is not the identification of a few powerful techniques, but rather the question of how to represent knowledge in a fashion that permits their effective use and interactions"

Many attempts to improve students' problem solving focuses on facts, concepts and procedures (see STRATEGIES section above). Erricson's (2003) extensive research on the development of expertise has shown that all experts have become so through deliberate practice and he further claims that experts have no special abilities that have allowed them to become experts.

It is easy to read this sort of literature and come away believing that facts, concepts and procedures (resources as Schoenfeld termed them) and strategies (heuristics) are all that is needed for effective problem solving. Observation of experienced problem solvers in physics (chi, Larkin, etc…) supports this idea. If the only definition of a problem were the tasks in the back of a physics text (exactly the type of task that was the focus of many classic studies in physics), then maybe one could believe that only knowledge is important when studying problem solving. But a close look at students when they first encounter new problems shows that there are many more skills needed when solving an unfamiliar task, since a task that is unfamiliar is an actual problem. In response to Goldstein and Papert's claim above, Simon (1980) says,

> "The error in this claim lies in its either-or stance. Two-bladed scissors are still the most effective kind. In addition to the large body of knowledge that is represented in semantically rich systems, there have to be processes for operating on that knowledge to solve problems and answer questions."

Schwartz, Bransford and Sears (2005) define two types of problem solving practice, innovation and efficiency. Efficiency means teaching rote problem solving. It's a focus on making certain skills automatic. Innovation requires students to attempt to create their own solution to a completely unfamiliar problem. Efficiency training is important because the issue of cognitive load is always looming (Ward and Sweller, 1990; Mayer, 2003) If a student is focusing on using new resources (facts, concepts and procedures) there is likely little if any cognitive space available for processing. So efficiency training is important and creating a solid knowledge base is imperative to problem solving but it is not everything. Innovation problems focus on practicing other important cognitive processes needed for problem solving. Schoenfeld (1985) describes control (processing) as the behavior that deals with the way students use their resources and heuristics. It focuses on major decisions about what to do in a problem. At the most basic level, how does a student decide which facts, concepts and procedures apply to a certain problem if they've never solved that particular problem?

Cognitive processing is known by many names; managerial decisions in business, executive decisions in Artificial Intelligence, strategic (vs. tactical) in the military or metacognitive in psychology. Pretz, Naples and Sternberg (2003; Sternberg, 1985) discuss the importance of meta-components which are needed to guide problem solving by planning, monitoring and evaluating the problem-solving process. These include 1.) recognizing the existence of a problem, 2.) defining the nature of the problem, 3.) allocating mental and physical resources to solving the problem, 4) deciding how to represent information about the problem, 5) generating the set of steps needed to solve the problem, 6) combining these steps into a workable strategy for problem solution, 7) monitoring the problem-solving process while it is ongoing, and 8) evaluating the solution. Pretz et al. delve into problem formation, how does a solver identify a problem? Problem definition is typically not required in the classroom. When looking at real problems in this manner it becomes even clearer how important processing is to solving problems. Much of classic psychology including Sternberg et al clumps all cognitive processing into metacognitive skills including Mayer (1998). More recent work (Mayer and Whitrock, 2006; Berardi-Coletta, Buyer, Dominowski and Rellinger, 1995) has recognized the distinction between processing skills of representation, planning/monitoring and executing that one engages in during problem solving and meta-processing. Meta-processing, as implied by its name, is reflection on one's processing.

The short version: Without the knowledge needed for the problem, it doesn't matter what amazing processing a person is capable of, they won't solve the problem without the necessary facts; and, If a person knows all the facts but comes across something slightly unfamiliar, they have to think whether they like it or not. So it takes both and neither is more important.

Ross (2007) offers an example not only the import of processing but of the value of dividing processing into its component skills. He states that a common problem when teaching is,

> "…a failing student would express dismay since s/he had understood the way to solve the problem perfectly when I explained it in class. I had to point out that the exam did not test their understanding of my solution, but their ability to generate their own solution. This mismatch in processing is a common and underappreciated influence on transfer."

Summary

Through this literature review I have found that a lot of research on problem solving in physics focuses on the dichotomy between experts and novices. Most of the results of this expert/novice work in physics can offer some information about knowledge structures and some on beliefs; however, very few studies use in-depth problems that require a variety of cognitive processing or consider beliefs, expectations and/or motivation in their analysis. Thus it is not surprising that nearly all of the attempts to improve problem solving focus on one correct strategy and/or ways to organize your knowledge. However, there is work in other disciplines' that does include the study of processes and beliefs (Schoenfeld, 1985; Wineburg, 1998) while solving difficult problems. When looking carefully at both research in physics and in cognitive science as well as other disciplines, the results do fit when one views problem solving as being comprised of many separate component skills.

To study problem solving carefully it is useful to consider three important characteristics 1.) you must have enough subjects to draw general inferences from your data; 2) The specific capabilities of your subjects must not be assumed (for example, using the person's level in school as an indication of problem solving ability); and 3) All subjects must be engaged in the same activity – the same level of knowledge retrieval and cognitive processing – if comparisons are to be made.

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