Quiz 10

Names:

Solution

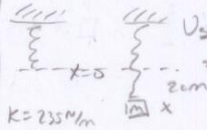
1. Explain what these three very similar equations are for. Explain a situation appropriate for each equation and show how it would be applied, what it represents and why in each case it's energy.

$K = \frac{1}{2} m v^2$

This first one is Kinetic Energy. Energy of motion which depends on the mass and the velocity of the particular object.
A 70kg jogger runs at 4 m/s. His Kinetic energy is $\frac{1}{2}\,70\,kg(4\,m/s)^2 = 560\,J$
This is specifically translational kinetic energy because it's for an object moving with a linear velocity.

$U_s = \frac{1}{2} k x^2$

This second equation is elastic potential energy. k is the spring constant, the stiffness of the object. x is the distance the elastic material is stretched or compressed from equilibrium.



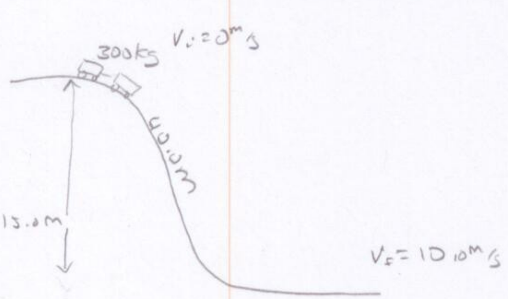
$U_s = \frac{1}{2}\,235\,N/m(0.02m)^2$
$= 0.047\,J$

The spring has the ability to do work based on the restoring force which will act over the distance x.

$K_r = \frac{1}{2} I \omega^2$

This equation is rotational kinetic energy which is an energy of motion (kinetic) specifically for rotating objects such as a wheel, marble, atom etc.... I is the moment of inertia which describes how difficult it is to change the motion of a rotating object. I depends on both mass and distance from the axis of rotation. ω is the angular velocity of the rotating object. This equation is the angular counterpart to $K = \frac{1}{2}mv^2$.
If a 1.0cm, 10g marble rotates at a speed of 18 rad/s its rotational kinetic energy is
$\frac{1}{2}(\frac{2}{5}\,.01\,kg\,(.01m)^2)(18\,rad/s)^2 = 0.000065\,J$

2. A 300 kg roller coaster starts from rest at the top of a hill 15.0 m high. After rolling unassisted to the bottom of the hill, its velocity is 10.0 m/s. Find the average force of friction that acted on the car if it traveled a total distance of 40.0 m while going down the hill.



First I'll determine how much energy went into thermal energy.

$KE_i + U_{gi} = KE_f + U_{gf} + E_{TH}$

$0 + mgh_i = \frac{1}{2}mv_f^2 + 0 + E_{TH}$

$mgh_i - \frac{1}{2}mv_f^2 = E_{TH}$

$300kg \cdot 9.8\,m/s^2 \cdot 15.0m - \frac{1}{2}\,300kg\,(10.0\,m/s)^2 = E_{TH}$

$44{,}100\,J - 15{,}000\,J = E_{TH}$

$29{,}100\,J = E_{TH}$

Now I know that the Thermal Energy came from the work done by friction.

$W_f = E_{TH}$

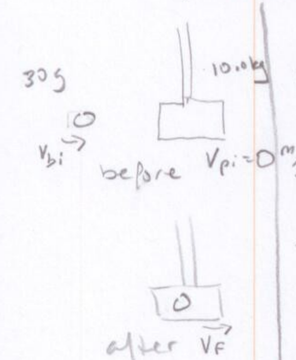
$F \cdot d = E_{TH}$

$F = \frac{E_{TH}}{d} = \frac{29{,}100\,J}{40.0m} = \boxed{728\,N}$

friction acts along the track so need the distance over which the friction acted.

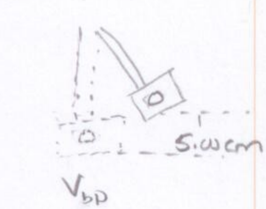
3. The ballistic pendulum was invented in 1742 by English mathematician Benjamin Robins (1707–1751), and published in his book *New Principles of Gunnery*, which revolutionized the science of ballistics, as it provided the first way to accurately measure the velocity of a bullet. It consists of a heavy iron pendulum faced with wood to catch the bullet. Based on the height the pendulum swings, the speed of the bullet is determined.

   A musket ball of 1 oz (30 g), is fired into a ballistic pendulum of mass 10.0 kg. If the pendulum swings to a point 5.00 cm higher than its resting position, what was the velocity of the musket ball just before it was embedded in the pendulum. (Hint: The answer is ***not*** 18 m/s, it's much faster.)



Collision

Swing



This is a perfectly inelastic collision which means that mechanical energy is not conserved but momentum is. You can apply conservation of momentum to the collision and use conservation of mechanical energy during the swing. It's easiest to work backwards. Use the swing to find the velocity of the bullet/pendulum combo, v_{bp}, just after the collision. Then use this as your final velocity of the collision

Swing

$\frac{1}{2}\cancel{M_{bp}} V_{bp}^2 = \cancel{M_{bp}} g h$

$V_{bp}^2 = 2gh$

$V_{bp} = \sqrt{2 \cdot 9.8\,m/s^2 \cdot .0500\,m}$

$= 0.9899\ m/s$

Collision

$m_b V_{bi} + m_p \cancel{V_{pi}}^{0\,m/s} = (m_b + m_p) V_{bp}$

$V_{bi} = \frac{m_b + m_p}{m_b} V_{bp}$

$= \frac{0.030kg + 10.0kg}{0.030kg}\ 0.9899\ m/s$

$\boxed{= 331\ m/s}$

$W = F\,\Delta x = \Delta E$ $\quad K = \frac{1}{2} m v^2$ $\quad U_g = mgh$ $\quad U_s = \frac{1}{2} k x^2$

$g = 9.8\ m/s^2$ $\quad K_r = \frac{1}{2} I \omega^2$ $\quad P = W/\Delta t = F v$

$\vec{p} = m\vec{v}$ $\quad \vec{F}\,\Delta t = \Delta\vec{p}$ $\quad \vec{p}_i = \vec{p}_f$ $\quad \Sigma\vec{F} = m\vec{a}$

$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$ $\quad a_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{\Delta t}$

$x_f = x_i + v_{xi}\Delta t + \frac{1}{2} a_x (\Delta t)^2$ $\quad v_{xf} = v_{xi} + a_x \Delta t$ $\quad v_{xf}^2 = v_{xi}^2 + 2a_x(\Delta x)$

$\sin\theta$ = opp/hyp $\quad \cos\theta$ = adj/hyp $\quad \tan\theta$ = opp/adj

$a^2 + b^2 = c^2$