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PROBLEM SOLVING

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A major challenge of education is improving students' minds—a goal that is reflected in people being able to solve novel problems they encounter. This is the premise underlying much of the interest in problem solving, including how to teach in ways that enable students to apply what they have learned to new situations and how to teach thinking skills. In this chapter, after defining key terms and providing a historical overview, we examine research on teaching for problem-solving transfer and research on teaching of thinking skills.

WHAT IS PROBLEM SOLVING?

Definitions

When you are faced with a problem and you are not aware of any obvious solution method, you must engage in a form of cognitive processing called *problem solving*. Problem solving is cognitive processing directed at achieving a goal when no solution method is obvious to the problem solver (Lovett, 2002; Mayer, 1992). According to this definition, problem solving has four main characteristics. First, problem solving is *cognitive*, that is, it occurs internally in the problem solver's cognitive system, and can only be inferred indirectly from the problem solver's behavior.[1] Second, problem solving is a *process*, that is, it involves representing and manipulating knowledge in the problem solver's cognitive system. Third, problem solving is *directed*, that is, the problem solver's cognitive processing is guided by the problem solver's goals. Fourth, problem solving is *personal*, that is, the individual knowledge and skills of the problem solver help determine the difficulty or ease with which obstacles to solutions can be overcome. Thus, problem solving is cognitive processing directed at transforming a given situation into a goal situation when no obvious method of solution is available (Mayer, 1990).

Related terms such as *thinking*, *reasoning*, *creative thinking*, and *critical thinking* are sometimes used as synonyms. In this chapter, we use *problem solving*, *thinking*, and *reasoning* interchangeably, although it is possible to make finer distinctions. For example, in the strictest sense, *thinking* refers to a somewhat broader concept that includes both directed cognitive processing (i.e., problem solving) and undirected cognitive processing (e.g., day dreaming). In this chapter, we focus only on directed cognitive processing. *Reasoning*, in the strictest sense, refers to directed cognitive processing applied to a certain class of tasks—that is, reasoning tasks in which there are premises and the goal is to derive a conclusion using logical rules—and requiring a certain class of cognitive processes—that is, deduction and induction

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[1] Although we have defined problem solving as an internal process, it can be aided and influenced by creating external representations, manipulating concrete objects, and interacting with others.

(Manktelow, 1999). In this chapter, we maintain a broader focus on all forms of directed cognitive processing. Finally, problem solving can be broken into creative thinking and critical thinking (Runco, 2003). Creative thinking involves generating ideas that could be used to solve a problem, whereas critical thinking involves evaluating ideas that could be used to solve a problem. In this chapter, we focus on both aspects of problem solving.

A problem occurs when a problem solver wants to transform a problem situation from the given state to the goal state but lacks an obvious method for accomplishing the transformation. In his classic monograph, *On Problem-Solving*, Duncker (1945, p. 1) defined a problem as follows:

> A problem arises when a living creature has a goal but does not know how this goal is to be reached. Whenever one cannot go from a given situation to the desired situation simply by action, then there has to be recourse to thinking. Such thinking has the task of devising some action, which may mediate between the existing and desired situations.

In short, a problem occurs when a problem solver has a goal but lacks an obvious way of achieving the goal. This definition is broad enough to include many high-level academic tasks such as writing a persuasive essay (Kellogg, 1994), solving an unfamiliar arithmetic word problem (Reed, 1999), or determining how an electric motor works (Mayer, Dow, & Mayer, 2003). It is also broad enough to include many high-level nonacademic tasks such as determining how to get 3/4 of 2/3 of a cup of cottage cheese (Lave, 1988) or determining which is the best apartment to rent (Kahneman & Tversky, 2000).

Types of Problems

Problems can be classified as well defined or ill defined. In well-defined problems, the given state, goal state, and allowable operators are clearly specified. For example, a computation problem such as 1.27 × 0.28 = __ is well defined because the given state is 1.27 × 0.28, the goal state is a numerical answer that is the product of 1.27 and 0.28, and the allowable operators are the procedures of decimal multiplication. Similarly, a grammar problem such as "the plural of half is __" is well defined because the given state is "half," the goal state is to create a specific word that is the plural form of "half," and the allowable operators are the procedures for constructing plurals in English grammar, namely, to change *f* to *v* and add the suffix *es*. In an ill-defined problem, the given state, goal state, and/or allowable operators are not clearly specified. For example, an assignment to write an essay on whether a tax cut will stimulate the economy or to devise an advertising campaign for the campus bookstore is an ill-defined problem because the allowable operators are not clear, and to some extent, the goal state is not clear. Educational materials often emphasize well-defined problems, although most real problems are ill defined.

Problems can be classified as routine or nonroutine.[2] exercise A routine problem is one for which the problem solver already possesses a ready-made solution procedure. For example, if a student has learned the procedure for long division of whole numbers, then a new long-division problem represents a routine problem. In contrast, a nonroutine problem is one for which the problem solver does not have a previously learned solution procedure. For example, a young student who does not yet know all the addition facts may solve the problem 3 + 5 = __ as follows: "Take 1 from 5 and give it to the 3, 5 minus 1 is 4, 3 plus 1 is 4, 4 plus 4 is 8, so the answer is 8." This student has invented a solution method that is new for the student. Thus, the definition of routine or nonroutine problem depends on the knowledge of the learner, whereas the definition of well-defined or ill-defined problem does not.[3] Although routine problems form the core of many educational lessons, important real-world problems are generally nonroutine.

Cognitive Processes in Problem Solving

Problem solving can be analyzed into [illegible] component cognitive processes, including representing, planning/monitoring, executing, and self-regulating. Representing occurs when a problem solver converts an externally presented problem, such as a word problem in a mathematics book, into an internal mental representation, such as a *situation model* of the word problem—that is, a representation of the situation being described in the problem (Mayer, 2003; Nathan, Kintsch, & Young, 1992). In classic theories of problem solving, representing a problem involves building a problem space—a representation of the initial state, goal state, and all legal intervening states (Bruning, Schraw, Norby, & Ronning, 2004). Planning involves devising a method for solving a problem, such as breaking a problem into

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[2] In the strictest sense, problem solving involves only non-routine problems; although in conventional usage, problem solving involves both non-routine and routine problems. We focus on non-routine problem solving in this chapter.

[3] The definition of ill-defined or well-defined problem does not depend on the problem-solver's knowledge because being ill or well defined depends on the characteristics of the problem. However, problem solvers can differ in their knowledge of the characteristics of the problems.

parts, whereas monitoring involves evaluating the appropriateness and effectiveness of the solution method. Executing occurs when a problem solver actually carries out the planned operations, such as making arithmetic calculations to solve a word problem. Self-regulating refers to instigating, modifying, or sustaining cognitive activities oriented toward the attainment of one's goals (Schunk, 2003), such as deciding to start over when having difficulty with a problem. Although executing is sometimes emphasized in classroom instruction, the major difficulties for most problem solvers involve representing, planning/monitoring, and self-regulating (Mayer, 2003).

Problem-solving processes are dependent on several different kinds of knowledge, including factual knowledge, conceptual knowledge, procedural knowledge, strategic knowledge, beliefs, and metacognitive knowledge. Factual knowledge involves knowledge of facts such as "there are 100 cents in a dollar" or "Washington is the capital of the United States." Conceptual knowledge includes knowledge of categories, principles, and models, such as the cause-and-effect explanation for why hot air rises or knowing how the Electoral College works in U.S. presidential elections. Procedural knowledge involves knowledge of specific procedures for how to do something, such as the procedure for long division or how to change nouns from singular to plural form. Strategic knowledge involves knowledge of general methods, such as how to break a problem into parts or how to summarize a passage. Metacognitive knowledge involves awareness and control of one's own cognitive processing and includes beliefs such as, "I am not good at math." Bruning et al. (2004) use the term *conditional knowledge* to refer to knowing when and why to use existing conceptual and procedural knowledge. The cognitive process of representing depends largely on facts and concepts; the cognitive process of planning/monitoring depends largely on strategies; the cognitive process of executing depends largely on procedures; the cognitive process of self-regulating depends on beliefs and related metacognitive knowledge. The types of processes and knowledge involved in problem solving are summarized in Table 13.1.

p. 13 [illegible] meta skills

TABLE 13.1. Types of Cognitive Processes and Knowledge Involved in Problem Solving

Process	Knowledge
Representing	Facts Concepts
Planning/monitoring	Strategies
Executing	Procedures
Self-regulating	Beliefs/metacognitive knowledge

Problem Solving as an Educational Goal

The primary goal of education is to promote learning—that is, a change in the learner's knowledge (Mayer, 2001). However, learning can only be indirectly assessed by observing changes in the learner's behavior, such as test performance. Two classic measures of learning outcomes are retention and transfer (Anderson et al., 2001). Retention is the ability to remember what was presented and can be assessed using recall and recognition items. Transfer is the ability to use what was learned in new situations and can be assessed using a variety of problem-solving items.

For example, in assessing a student's learning of a lesson on how an electric motor works (e.g., Mayer et al., 2003), a retention item could ask the student to write definitions of each of the key components, such as "battery" and "magnet," that were defined in the lesson. In contrast, a transfer item could ask the student to figure out how to reverse the direction of the rotation of the motor, which was not directly presented in the lesson but could be determined by understanding how the motor works. A student who performs poorly on both types of assessments has a learning outcome that can be called *no learning*. A student who performs well on retention and poor on transfer has achieved a learning outcome that can be called *rote learning*. A student who performs well on both kinds of assessments has achieved a learning outcome that can be called *meaningful learning*.

In cases where the instructional objective is to promote meaningful learning rather than rote learning, it is useful to understand the nature of problem solving. Although helping students remember what was presented has been a central goal of education, there is consensus among educational psychologists that another important goal is helping students be able to apply what they learned to solve new problems (Anderson et al., 2001).

INSTRUCTIONAL METHODS THAT PROMOTE PROBLEM SOLVING

What are instructional methods that promote meaningful learning? Table 13.2 lists seven ways of promoting meaningful learning: load-reducing methods, structure-based methods, schema activation methods, generative methods, guided discovery methods, modeling methods, and teaching of thinking skills. The first six methods aim to help students learn content in ways that promote problem-solving transfer,

TABLE 13.2. Seven Ways to Promote Problem-Solving Transfer

Instructional Method	Example
Load-reducing methods	Automaticity, constraint removal
Structure-based methods	Concrete manipulatives
Schema-based methods	Advance organizers, pre-training, cueing
Generative methods	Elaboration, note-taking, self-explanation, questioning
Guided discovery methods	Guided discovery
Modeling methods	Worked examples, apprenticeship
Teaching thinking skills	General courses, specific strategies

which can be described as promoting problem solving in a domain; the seventh method aims to teach problem-solving skills directly, which can be described as promoting general problem-solving abilities. In the remainder of this chapter, after a short historical example and theoretical discussion, we explore the seven approaches to fostering problem solving listed in Table 13.2.

Example of Instructional Methods That Promote Problem Solving

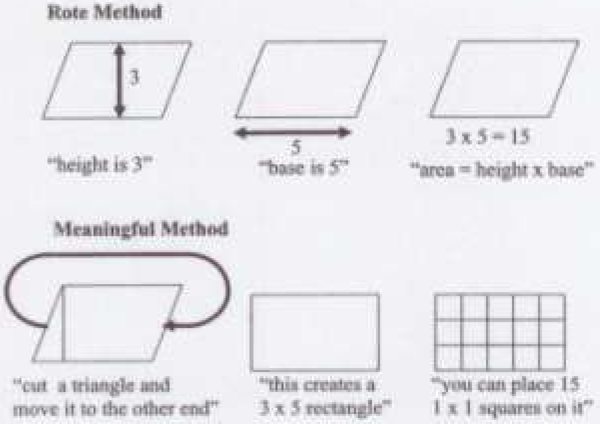

FIGURE 13.1. Two methods for teaching students how to find the area of a parallelogram.

The search for methods of instruction that lead to meaningful learning—that is, learning that enables students to apply what they have learned to solving new problems—has a long, and increasingly productive history in educational psychology. For example, in his classic book *Productive Thinking*, the Gestalt psychologist Max Wertheimer (1959) described two ways of teaching children how to find the area of a parallelogram. In the rote method, summarized in the top of Fig. 13.1, students were taught to find the height (i.e., 3), find the base (i.e., 5), and then multiply height *x* base (i.e., 15), using the formula Area = Height × Base. Wertheimer calls this method senseless, blind, and arbitrary. In the meaningful method, summarized in the bottom of Fig. 13.1, students were encouraged to see that the triangle on one end of the parallelogram could be cut off and placed on the other end of parallelogram, resulting in a rectangle. As can be seen, placing 1×1 squares on the rectangle yields three rows of five squares, for a total of 15. Assuming that the problem solver already knows how to find the area of a rectangle, the solution is now apparent. Wertheimer argues that this method leads to structural insight and deep understanding. On a retention test in which students must find the area of similar parallelograms, both forms of instruction lead to good performance (Wertheimer, 1959). However, on a transfer test in which students must find the area of unusual parallelograms and other figures, the meaningful method leads to much better problem-solving performance than does the rote method. Thus, according to Wertheimer and other Gestalt-oriented psychologists (Katona, 1967), the distinguishing feature of meaningful methods of instruction is that they enable students to apply what they have learned to solve new problems.

This method

Theoretical Foundations for Instructional Methods That Promote Meaningful Learning

According to the Select-Organize-Integrate (SOI) information-processing theory of meaningful learning summarized in Fig. 13.2, meaningful learning occurs when learners engage in three cognitive processes during learning: selecting relevant information, organizing the selected information, and integrating the organized information with prior knowledge (Mayer, 2001, 2003). The SOI model in Fig. 13.2 contains three memory stores indicated by boxes—sensory memory, working memory, and long-term memory—and three main cognitive processes indicated by arrows—selecting, organizing, and integrating. When material is presented to a learner, such as in a book or lecture, the pictures and printed words impinge on the learner's eyes and are represented as images in sensory memory, whereas the spoken words impinge on the learner's ears and are represented as sounds in sensory memory. If the learner pays attention, some of the incoming sounds and images in sensory memory are transferred to working memory for further processing, as indicated by the arrow labeled *selecting*. In working memory, the learner can build connections among the pieces of selected information, as indicated by the arrow labeled *organizing*, and the learner can build connections between the selected information and relevant knowledge retrieved from long-term memory,

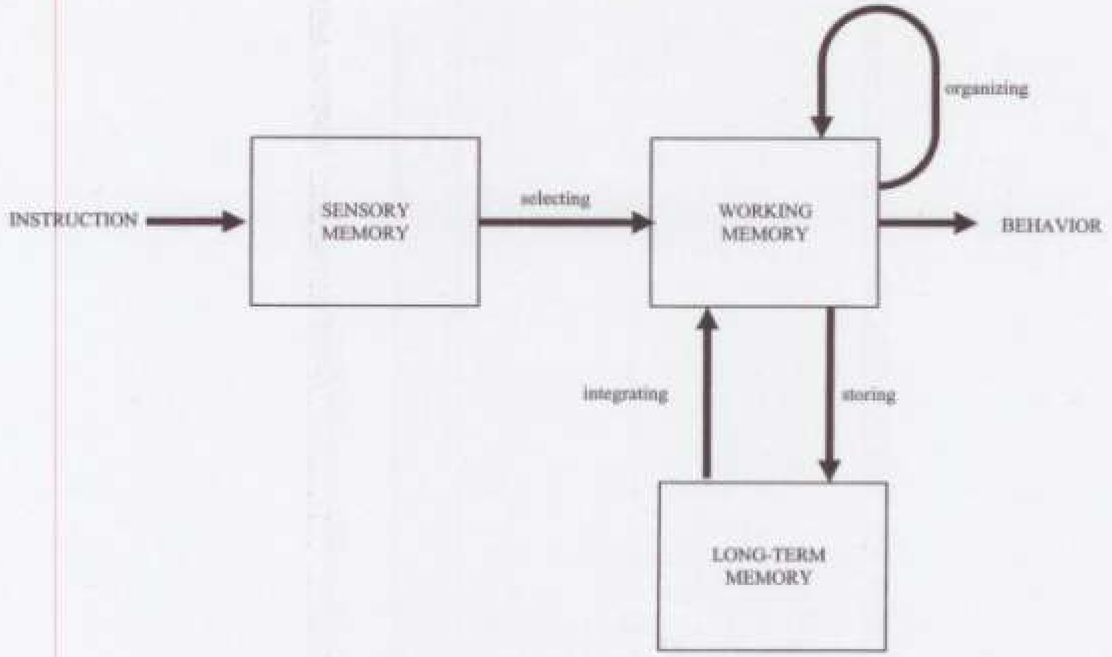

FIGURE 13.2. The SOI model of meaningful learning.

as indicated by the arrow labeled *integrating*. The result is a coherent cognitive structure that itself can be stored in long-term memory for future use, as indicated by the *storing* arrow.

A major implication of the SOI model is that three cognitive conditions must be met for meaningful learning to occur: the learner must select relevant information from the presented material, the learner must mentally organize the material into a coherent structure, and the learner must mentally integrate the organized knowledge with existing knowledge retrieved from long-term memory. Instructional methods that foster these cognitive processes—selecting, organizing, and integrating—are more likely to lead to meaningful learning—and hence, problem-solving transfer—than are methods that do not foster these processes. For example, in the parallelogram problem, the meaningful method of instruction helps learners (a) select the critical structural features of a parallelogram, namely the triangles on each end; (b) organize the parallelogram as a rectangle in disguise; and (c) integrate the rearranged parallelogram with prior knowledge about area of rectangles.

Load-Reducing Methods

The SOI model in Fig. 13.2 shows that cognitive processing takes place in working memory, but working memory capacity is limited. In problem solving, low-level processes involved in executing a solution—such as carrying out arithmetic calculations in solving a word problem—may require so much working memory capacity that the problem solver does not have capacity left for devising and monitoring a solution plan. Therefore, effective problem solving often depends on automated component skills—that is, on component skills that can be used without placing demands on the student's working memory capacity. In short, a major constraint on effective problem solving may be lack of automated component skills. Two methods for overcoming this problem are automaticity methods, in which students master component skills before moving on to higher level problem tasks, and constraint removal methods, in which problem-solving tasks are presented in ways that minimize the need for certain component skills.

Automaticity Methods. In automaticity methods, students receive drill and practice on component skills until the skills have become automatic. Automated skills require little or no conscious attention when they are applied; since attention is a limited resource, automation of low-level skills allows problem solvers to direct their attention to higher-level components of a task such as planning and monitoring the solution process. Classic work on learning hierarchies by Gagné (1968) and subsequent research on the development of problem-solving skills in children by Case (1985) and Siegler (1989) have shown how automation of low-level component skills enables the learner to build higher level problem-solving skills.

For example, reading comprehension depends on mastering lower level skills such as decoding of words. To help students become more automatic in their decoding, Samuels (1979) and LaBerge and Samuels (1974) developed the method of repeated readings, in which a student reads a short passage aloud over and over until the reading rate is fast and the error rate is low. This procedure allows students to increase their fluency, that is, to become automatic in their decoding of passages. As students automate their decoding skills, they can devote more attentional capacity to comprehending the passage. Similarly, young students who were given practice in phonological awareness—recognizing and producing the sounds in English—subsequently showed much greater performance on tests of reading comprehension than did nontrained students, and the advantage persisted for many years (Bradley & Bryant, 1983, 1985).

In mathematics, solving equations in algebra depends on component skills such as collecting all the unknowns on one side of the equation, and generating proofs in geometry depends on component skills such as recognizing when two angles are congruent. Anderson and colleagues (Singley & Anderson, 1989) have shown how the acquisition of cognitive skills progresses from effortful performance requiring conscious attention to automatic performance that does not require attention. For example, computer-based tutors for algebra, geometry, and computer programming provide systematic practice on component skills that eventually become automated and can be combined to form more powerful skills (Anderson & Schunn, 2000; Anderson, Corbett, Koedinger, & Pelletier, 1995). By performing a cognitive task analysis and providing systematic practice in the low-level component skills, "students can achieve the same level of competence in one third of the time as traditional education" (Anderson & Schunn, 2000, p. 26). Thus, the development of problem-solving expertise in algebra, geometry, and computer programming was facilitated by providing systematic practice in the lower-level component skills.

What to doing efficiently

Similarly, Ericsson (2003a, 2003b) has shown that the development of expert problem-solving performance in fields such as music and sports is strongly related to the amount of *deliberate practice*—systematic and sustained practice on the elements of the task. For example, case studies of students designated as talented musicians revealed that a major difference between those who excelled and those who did not was the amount of deliberate practice accumulated during their development, with the most elite group estimated to have spent more than 10,000 hours in solitary practice by the age of 20 (Ericsson, 2003a). Thus, it appears that a prerequisite for excellence in creative performance is that basic skills have been well practiced.

Constraint Removal Methods. If automaticity methods were the only ones available, early childhood education would not provide many opportunities to engage in problem solving. An alternative method that allows novices to gain problem-solving experience, and presumably enjoyment, in a new domain is constraint removal—that is, creating problem-solving tasks that do not require attention-demanding skills. For example, in writing an essay, young writers may lack automated motor skills (such as handwriting or typing) and automated grammatical skills (such as in spelling and punctuation). To remove these motoric and grammatical constraints on effective writing, students may be asked to dictate their essays (Scardamalia, Bereiter, & Goelman, 1982) or instructed not to worry about handwriting, spelling, and punctuation as they write (Glynn, Britton, Muth, & Dugan, 1982). Such methods allow students to devote more of their attention to organizing and planning a good essay and generally result in longer and better-written essays.

In mathematics, students may be asked to solve complex word problems before they have mastered basic computational skills. To remove this computational constraint, students can be given calculators. In a meta-analysis of 88 studies, Hembree and Dessart (1992) found that mathematical problem solving was improved by the use of calculators for all ability levels and in all grade levels. In a recent review of research on calculator use, Kilpatrick, Swafford, and Findell (2001, p. 427) concluded that "a large number of empirical studies of calculator use. . . have generally shown that the use of calculators does not threaten the development of basic skills and that it can enhance conceptual understanding, strategic competence, and disposition towards mathematics." Importantly, the authors note that students who are allowed to use calculators "are better able to tackle realistic mathematics problems" (p. 427), presumably because they can allocate their limited attentional resources to high-level processes such as representing, planning, and monitoring rather than to trying to carry out low-level arithmetic procedures.

Structure-Based Methods

According to the SOI model in Fig. 13.2, meaningful learning depends on active cognitive processing during learning, including selecting, organizing, and integrating. One way to prime active cognitive processing in learners is to use structure-based methods, in which the learner is given concrete objects that can be manipulated. A primary purpose of manipulative methods is to help the learner build connections between a familiar, concrete situation and a more abstract concept or rule. For example, in mathematics instruction, concrete manipulatives such as bundles of sticks, beads on sticks, or Dienes blocks can be used to help the learner understand simple computational procedures (Brownell & Moser, 1949; Dienes, 1960; Montessori, 1964).

More recently, computer-based microworlds have been used in which students who are allowed to manipulate objects on the screen learn to solve mathematics problems better than students who do not (Moreno & Mayer, 1999; Nathan, Kintsch, & Young, 1992). For example, students who could move a bunny along a number line on the computer screen in correspondence with addition and subtraction of signed numbers learned to solve problems better than students who were not given this experience (Moreno & Mayer, 1999), and students who could construct a pictorial representation of word problems on a computer screen learned to solve word problems better than students who did not have this experience (Nathan, Kintsch, & Young, 1992).

In science instruction, hands-on activities are used in which students make predictions and then participate in an actual experiment to test the prediction (Linn & Hsi, 2000). In addition, structure-based methods have been applied successfully to computer-based learning environments, such as White's (1993; White & Frederiksen, 1998) Thinker Tools computer game for teaching Newton's laws of motion.

More than two decades ago Resnick and Ford (1981) noted that there is surprisingly little research on the effects of manipulatives on student learning. Learners may have difficulty in making connections between internal and external representations because such efforts place heavier demands on working memory, or learners may lack the metacognitive skills for self-directed manipulation of concrete objects or in microworlds (de Jong & van Joolingen, 1998). More recent studies have attempted to discriminate situations in which manipulatives are most likely to be helpful (Fuson & Briars, 1990; Hiebert et al., 1997). For example, Kilpatrick, Swafford, and Findell (2001, p. 354) note that manipulatives may overload working memory when they "become one more thing to learn" but "when used well, manipulatives can enhance student understanding." English (1997, p. 4) argues that manipulatives reflect "a move away from the traditional notion of reasoning as primarily propositional, abstract, and disembodied to the contemporary view of reasoning as embodied and imaginative." Overall, using concrete manipulatives has potential for fostering meaningful learning under appropriate conditions.

Schema-Activation Methods

In schema-activation methods, instruction either provides or activates relevant prior knowledge. Schema-activation methods encourage and enable the learner to connect new incoming material with relevant existing knowledge (as indicated by the integrating arrow in Fig. 13.2) and are especially effective for novices (Mayer, 2003). Three forms of schema-activation methods are advance organizers, pretraining, and cueing.

Advance Organizers. An advance organizer is material presented before a lesson that is intended to promote learning by helping the learner relate the new material to existing knowledge. Ausubel (1968) described how advance organizers could help learners assimilate incoming information to their existing knowledge. For example, Mayer (1983) asked students to read a passage on radar and take a problem-solving test on the material. Before reading the passage, some students received a labeled diagram showing that radar was like throwing a ball, having it hit a remote object, and measuring the time it takes to return. The purpose of the diagram was to help students think of radar in terms of their prior knowledge about bouncing objects. Students who received the diagram performed much better on the problem-solving test than students who did not receive the diagram. In a review of research on advance organizers, Corkill (1992) found consistent support for advance organizers, although concrete organizers tended to be more effective than abstract ones.

Pretraining. Another approach is to provide some pretraining that familiarizes the learner with the elements described in the lesson. For example, Mayer, Mathias, and Wetzell (2002) asked students to view a narrated animation explaining how a pump works and then take a problem-solving test. Before the lesson, some students were given a clear plastic model of a pump and told to pull the handle up and down a few times. Students who had this short pretraining performed much better on the problem-solving test than did students who had not received the pretraining. Presumably, the experience of seeing and using a concrete pump helped activate relevant prior knowledge about pistons in cylinders that was useful for making sense of the narrated animation.

Cueing. Finally, a third approach is to alter a text passage so that it cues the reader to think of relevant prior knowledge. For example, adding illustrations to a text about how pumps work greatly increased subsequent problem-solving performance (Mayer & Gallini, 1990), presumably because the concrete illustration helped people use their prior knowledge about a piston moving through a cylinder. Similarly, Beck, McKeown, Sinatra, & Loxterman (1991) rewrote a history text on the French and Indian War that was particularly difficult for students to understand. For example, they added a first sentence stating "About 250 years ago, Britain and France both claimed to own some land, here, in North America" (p. 257). This sentence was intended to "activate the conflict schema in the reader's mind" (p. 257), so students could understand that two groups wanted the same object. Consistent with predictions, the rewritten version resulted in much better performance on subsequent tests of problem solving.

Generative Methods

Generative methods require that learners generate relations between their existing knowledge and information to be learned, as indicated by the *integrating* arrow in Fig. 13.2. Four forms of generative methods are elaboration, summary note-taking, self-explaining, and questioning.

Elaborative Methods. One form of generative method involves elaboration, in which the learner is explicitly asked to explain how the new material is related to existing knowledge. For example, asking elementary school students to construct verbal and imaginal relations between printed stories and their own experiences increased comprehension substantially compared with control groups that did not engage in these elaboration activities (Linden & Wittrock, 1981). Among high school students, elaboration methods that led students to construct verbal and graphical relations between concepts in economics and their knowledge increased comprehension and transfer of economics principles (Kourilsky & Wittrock, 1987, 1992). Asking students to generate analogies as they read a text is intended to encourage them to construct connections between the new material and their existing knowledge; Wittrock and Alesandrini (1990) found that instructions to generate analogies increased comprehension.

Similarly, in a lesson on using a database programming language, some students were asked to write a short description of how each command related to a familiar situation such as sorting documents from an in-basket into two piles based on some criterion, whereas other students were not. Students who generated the descriptions relating each command to their existing knowledge about clerical work in an office performed much better on subsequent problem-solving tests in which they had to create or interpret new programs (Mayer, 1980). In elaborative interrogation, learners read a passage and answer "why" questions intended to help them make needed inferences (Wood, Pressley, & Winne, 1990). Generative methods have also been applied to mathematics learning (Peled & Wittrock, 1990; Wittrock, 1974a) and science learning (Osborne & Wittrock, 1983, 1985). Wittrock has presented a model of generative learning (1974a, 1990) and a model of generative teaching (1991).

Note-Taking Methods. Asking students to take summary notes on a textbook lesson or classroom lecture can encourage them to select relevant information, summarize it coherently, and relate it to their past knowledge (Kiewra, 1991; Peper & Mayer, 1978). For example, Peper and Mayer (1978) found that students who were required to take notes on a videotaped statistics lecture performed better on problem-solving tests than students who were not allowed to take notes. Similarly, Doctorow, Wittrock, and Marks (1978) reported that elementary school students who were asked to generate summaries of the paragraphs in a text increased their comprehension substantially compared to students who were not allowed to generate summaries. More recently, Thiede and Anderson (2003) found that students who were asked to write summaries of a text they had read earlier displayed better metacognitive accuracy in judging how well they understood the text, when compared to students who only read the text.

Self-explanation Methods. Another form of generative learning is to engage in a process of self-explanation while learning, such as trying to explain a text to oneself while reading it (Chi, 2000). For example, as they read a passage on the human heart and circulatory system, students are asked to "read each sentence out loud and then explain what it means to you" including "how does it relate to what you've already read, does it give you new insight into your understanding of how the circulatory system works, or does it raise a question in your mind . . ." (Chi, 2000, p. 171). Chi and colleagues (Chi, 2000; Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Chi, de Leeuw, Chiu, and LaVancher, 1994) have found that asking students to engage in self-explaining as they read a science text helped them understand the material and subsequently led to better problem solving performance.

Questioning Methods. Finally, as a fourth example of a generative method consider questioning methods, in which the learner is asked to generate a question for each segment of a textbook lesson or classroom lecture. Generating questions is a part of some effective reading comprehension problems (Palinscar & Brown, 1984) but is most effective when students are trained in how to ask useful questions (Pressley & Woloshyn, 1995). For example, King (1992, 1997) has shown that students learn more deeply when they are taught questions to ask and prompted to use them when learning new material.

Guided Discovery Methods

Another technique advocated for encouraging active learning is discovery. In discovery methods, a problem is presented for the student to solve. In pure discovery, no guidance is provided so learners can explore on their own. In guided discovery, the teacher provides enough guidance to ensure that the learner discovers the rule, principle, or concept that is the goal of instruction. In expository methods, the learner is simply told the correct rule, principle, or concept. According to the SOI model shown in Fig. 13.2, pure discovery methods can prime the integration process—in which the learner searches long-term memory for ideas—but may not prime the selecting and organizing processes—in which the learner comes into contact with the to-be-learned material. In expository methods, the selecting process is primed—so the learner is aware of the to-be-learned material—but the organizing and integrating processes may not be activated because the learner is not encouraged to make sense of the material. Finally, in the guided discovery method, enough guidance is provided to prime the selecting process, whereas enough freedom is allowed to prime the organizing and integrating processes, yielding a meaningful learning outcome.

One of the earliest tests of the discovery methods involved research on learning of problem-solving rules conducted through the 1960s. For example, Gagne and Brown (1961) asked students to derive formulas to solve series sum problems such as "1, 3, 5, 7, 9. . . ". Guided discovery methods resulted in better performance on subsequent problem-solving tests than did pure discovery or expository methods. Although discovery was advocated for many large-scale curriculum development projects in the 1960s (Bruner, 1961), a review of subsequent research showed that pure discovery was not as effective as guided discovery in promoting problem-solving transfer (Shulman & Kieslar, 1966).

Through the 1970s, one avenue of research on discovery methods involved helping children discover Piagetian (1970) conservation strategies needed to solve conservation problems. For example, Gelman (1969) found that the best way to help kindergarteners learn to solve conservation problems was to provide guided practice in which the teacher directed the children's attention to relevant aspects of the task and provided feedback on their answers. In a recent review, Brainerd (2003) noted that there is a lack of evidence to support the use of pure discovery methods to help children learn conservation strategies, whereas guided methods have been shown to be effective.

A third approach to the study of discovery methods involved research on teaching of the LOGO programming language conducted through the 1980s. Consistent with Papert's (1980) call for allowing children to learn LOGO by interacting with a computer, some early instructional programs focused on pure discovery as the method of instruction. However, Kurland and Pea (1985) found that pure discovery methods did not promote subsequent problem solving, whereas Fay and Mayer (1994) found that children learned LOGO better from a guided discovery method in which they received help in design principles for programs. In a review, Littlefield et al. (1988, p. 116) concluded that "mastery of the programming language has not been achieved when LOGO has been taught in a discovery-oriented environment," whereas guided methods have been successful.

In a recent review of these three research literatures, Mayer (2004a, p. 14) concluded that "there is sufficient research evidence to make any reasonable person skeptical about the benefits of discovery learning." Instead, consistent with the SOI model, guided discovery methods resulted in better performance on subsequent problem solving tests than did pure discovery. Mayer (2004a, p. 17) noted: "Activity may help promote meaningful learning, but instead of behavioral activity per se (e.g., hands-on activity, discussion, and free exploration), the kind of activity that really promotes meaningful learning is cognitive activity (e.g., selecting, organizing, and integrating knowledge)."

Modeling Methods

In modeling methods, an instructor demonstrates how to solve problems and, in some cases, provides an explanation of the solution steps. The goal is to help the learner make sense of the solution by mentally organizing it into meaningful chunks and relating it to relevant prior knowledge. Two forms of modeling methods are example methods, in which the learner receives worked-out examples, and apprenticeship methods, in which the learner works with others on an authentic task.

Example Methods. The goal of example methods is to show learners how to solve typical problems in a field or subject area. When confronted with a new problem (which can be called a *target problem*), the problem solver is expected to engage in three cognitive processes: *recognizing* that the target problem is like a problem the problem solver already knows how to solve (from a storehouse of *base problems* in the problem solver's long-term memory); *abstracting* a solution principle or method from the base problem; and *mapping* the principle or method back onto the target problem to aid in solving the problem (Quilici & Mayer, 1996). For example, Cooper and Sweller (1987) taught students how to solve algebra equation problems either by giving them practice in solving eight problems (learning by doing), or by giving four pairs of problems where the first one showed a step-by-step example of how to solve the problem and the second was to be solved by the learner using the same solution method (learning from examples). Students in the learning from examples group learned faster and showed better performance on solving new problems as compared to students in the learning by doing group. Paas and van Merrienboer (1994) reported similar results in teaching students to solve geometry problems.

More recent work has shown that students learn best when the worked examples clearly specify the subgoals that each set of steps accomplishes (Catrambone, 1998), when instruction is designed to fade from providing all of the steps to providing fewer and fewer of the steps and asking the learner to fill them in (Renkl & Atkinson, 2003), and when students receive structure-emphasizing training that helps focus their attention on the structural features of examples (i.e., the underlying relations among the elements) rather than the surface features (i.e., the cover story in the problem) (Quilici & Mayer, 1996). In a recent review, Renkl (2005) documents the features of effective examples based on research evidence.

Case-based learning is a more complex form of learning from examples, in which the learner is given a realistic problem scenario—that is, a case—and asked to analyze the solution method (Mayer, 2003). Although case-based learning (as well as problem-based learning) has led to some exciting development projects in professional training, there is a need for a coherent research base of scientifically rigorous evidence (Evensen & Hmelo, 2000; Lundeberg, Levin, & Harrington, 1999; Schank, 2002).

Apprenticeship Methods. Apprenticeship methods represent an extended form of learning by example in which novices work closely with more experienced performers on authentic tasks (Lave, 1988; Lave & Wenger, 1991). Collins, Brown, and Newman (1989) have shown that throughout most of human history, before schools appeared, apprenticeship was the most common method for teaching people to become experts in fields ranging from medicine to law to art. Apprenticeship includes *modeling*, in which a teacher describes her or his cognitive processes while carrying out a task; *coaching*, in which a teacher provides advice and assistance to a student who is carrying out a task; and *scaffolding*, in which the teacher performs or removes part of the task that a student is not able to perform (Collins, Brown, & Newman, 1989). Two ways to implement aspects of the apprenticeship method in classrooms are Brown and Palinscar's (1989) reciprocal teaching and Slavin's (1990) cooperative learning.

In reciprocal teaching, a group of students and a teacher work together on an authentic academic task such as comprehending a text passage. For example, in one study (Brown & Palinscar, 1989; Palinscar & Brown, 1984), students and teacher took turns in leading a dialogue about how to carry out comprehension strategies, such as questioning (i.e., generating a relevant question that the passage answers), clarifying (i.e., identifying problematic words or phrases), summarizing (i.e., producing a concise summary), and predicting (i.e., inferring what will occur next in the passage). Students who engaged in reciprocal teaching showed a much greater improvement in their reading comprehension scores than students who learned with conventional methods.

In cooperative learning, students work in groups to learn how to perform an authentic academic task (such as how to solve arithmetic problems) in which there are incentives to help all members of the group to learn (Slavin, 1990). In a year-long study comparing cooperative learning and conventional teaching methods for teaching mathematics, students in the cooperative learning group showed much greater improvement in mathematics achievement than did students in the conventional group (Slavin & Karweit, 1984).

Although apprenticeship methods such as reciprocal teaching and cooperative learning can be effective in some situations, there are also many cases in which learning in groups is not particularly effective, so it is important to determine the conditions under which apprenticeship methods work (Johnson & Johnson, 1990). The challenge of such instruction is to encourage active cognitive processing that includes all three of the processes shown in Fig. 13.2.

Teaching Thinking Skills

Each of the preceding six methods for promoting problem solving was based on teaching content in ways that would facilitate its usability in subsequent problem solving. A somewhat different approach is to directly teach problem-solving skills to students—an approach that can be called teaching thinking skills. Teaching of thinking skills is part of the hidden curriculum, in the sense that educators expect students to be able to solve problems using the material presented in the course but rarely provide problem-solving instruction. Making such instruction part of the regular curriculum represents the seventh method for promoting meaningful learning.

Teaching Thinking Skills Courses. Many thinking skills programs and courses have been developed to teach problem-solving skills that will transfer to new problems (Adams, 1989; Chance, 1986; Martinez, 2000; Perkins & Grotzer, 1997). Mayer (1997) has proposed four issues for thinking skills programs: what to teach (e.g., thinking as a single ability or as a collection of component skills), how to teach (e.g., focusing on product or process), where to teach (e.g., in a general, domain-independent course or in an existing, specific course), and when to teach (e.g., after basic skills are mastered or before). A comparison of successful and unsuccessful programs reveals that teaching thinking skills is most effective (1) when the curriculum focuses on one or more component skills, such as how to break a problem into parts or how to evaluate hypothesis against data, rather than on improving the mind in general; (2) when the instructional method focuses on problem-solving processes, such as having experts model their problem-solving steps, rather than solely on getting the right answer; (3) when students are expected to be able to solve problems in the same domain as in the instruction rather than across domains; and (4) when skills are taught even before students have automated the underlying basic skills.

For example, in one of the first studies on teaching of thinking skills, Bloom and Broder (1950) sought to teach college students how to solve examination problems in subjects such as economics. As part of the training, students listened to successful problem solvers describe their thought processes as they solved examination problems, students described their own processes on the same problems, and students noted the differences between how they and the experts solved the problems. This training resulted in significantly higher scores on the examination than for students who did not receive training. Consistent with three of the criteria for successful instruction in thinking skills, the training focused on component skills, such as how to break a problem into parts; emphasized process, such as comparing one's own method with that of an expert; and was taught within a context that was similar to the final test. However, in terms of the fourth criterion, students already had solid knowledge of the fundamentals of economics, so in this sense the basic skills were already strong.

Perhaps the most studied thinking skills course is the Productive Thinking Program, a problem-solving course for elementary school children (Covington, Crutchfield, & Davies, 1966; Covington, Crutchfield, Davies, & Olton, 1974). The Productive Thinking Program consists of 15 cartoon-like booklets that describe various mystery or detective stories in which two children, Lila and Jim, try to solve the case. Throughout the booklet, the reader is asked to generate hypotheses, find relevant information to test the hypotheses, and engage in other problem-solving activities. Students who take the course tend to show larger pretest-to-posttest gains than control students on solving problems like those in the booklet, but "there is only limited . . .evidence of transfer to dissimilar problems" (Mansfield, Busse, & Krepelka, 1978, p. 531). Overall, this program meets the four criteria of teaching component skills (such as how to generate and evaluate hypotheses), teaching by modeling correct problem-solving processes (such as through the characters in the booklet), promoting problem-solving performance on problems like those in the booklet, and teaching high-level skills to children who had not yet automated their low-level skills.

Another well-documented attempt to teach thinking skills is Feuerstein's Instrumental Enrichment program (1980; Feuerstein, Hensen, Hoffman, & Rand, 1985). Students who were labeled as mentally retarded based on traditional tests of intelligence were given problem-solving classes several times per week over the course of several years. In a typical lesson, the teacher presented an unfamiliar problem, asked the students to work on it, and then led a class discussion in methods for solving the problem. In this way students could compare their thought processes with those of others. Evaluation studies revealed that students in the Instrumental Enrichment program showed greater pretest-to-posttest gains on tests of nonverbal intelligence than students given conventional instruction. Although the program appears to be effective, Chance (1986, p. 85) pointed out that it "requires a considerable investment of student time," and Bransford, Arbitman-Smith, Stein, and Vye (1985, p. 201) observed that the program emphasizes "training students to solve certain types of problems so they will be able to solve similar problems on their own." Again, the program meets the criteria of teaching component skills, focusing on problem-solving process, promoting performance within the same domain, and teaching young students who have not yet mastered basic skills.

Finally, Odyssey was designed as a middle-school course intended to increase students' performance on intelligence tests (Adams, 1989; Nickerson, 1994). Odyssey consists of approximately one hundred 45-minute lessons on basic problem-solving tasks such as how to solve a series completion problem—a problem sometimes found on intelligence tests. Each lesson begins with the teacher introducing the problem to the class, students work on the problem on their own, and students are asked to demonstrate and justify their solutions to the class. An evaluation of the project found that students who received Odyssey training for a year showed greater improvements than non-Odyssey students on a battery of cognitive tasks similar to those in the training. Overall, the training "enhanced the magnitude of students' intelligent behavior [on] authentic tasks at least in the short term" (Grotzer & Perkins, 2000, p. 496). The course targets component skills, focuses on modeling of appropriate problem-solving processes, improves performance mainly in the target domain, and works even when students have not fully mastered basic skills.

Overall, Mayer (2003, p. 425) has abstracted four general guidelines for the design of problem-solving programs, based on the features of successful programs: "1. Focus on a few well-defined skills. 2. Contextualize the skills within authentic tasks. 3. Personalize the skills through social interaction and language-based discussion of the process of problem solving. 4. Accelerate the skills so that students learn them along with lower-level skills." Although problem-solving courses have been popular for more than 50 years, the newer generation of thinking skills instruction focuses instead on teaching relevant cognitive skills within the context of specific subjects (Mayer, 2003). In the next sections, we explore teaching of cognitive and metacognitive skills needed for problem solving in various domains.

Teaching Domain-Specific Thinking Skills. Given the consistent evidence that thinking skills courses promote transfer mainly to similar problems within the same domain, a reasonable approach is to incorporate thinking-skills instruction within specific subjects such as mathematical, scientific, historical, and literary problem solving. In taking a domain-specific approach, the starting point is finding an authentic academic task and determining the main underlying cognitive processes through a cognitive task analysis.

For example, the academic task of writing an essay can be broken down into three major cognitive processes—planning, translating, and reviewing (Hayes, 1996; Hayes & Flower, 1980). Concerning the teaching of planning skills, students wrote better essays when they were required to spend time outlining their essay before they started writing (Kellogg, 1994). As part of a larger project, Englert, Raphael, Anderson, Anthony, & Stevens (1991) taught planning skills by asking elementary school children to fill out "plan-think-sheets" in which they answered questions such as, "Who am I writing for?", "Why am I writing this?", "What do I know?", and "How can I organize my ideas?" Filling out sheets that encouraged each of the main cognitive processes in writing resulted in increased quality of essays as compared to a group that did not receive training in planning skills and other component skills for essay writing. Similarly, Harris and Graham (1992) devised a set of planning questions to help students learn appropriate planning skills in writing stories, such as "Who is the main character?", "Where does the story take place?", and "What does the main character do or want to do?" Students who learned to plan using these kinds of questions showed greater improvement in the quality of their stories than students who did not receive such training. Overall, it appears that it is possible to improve students' planning skills by giving them directed practice in how to plan their writing.

As another example, the academic task of solving an arithmetic word problem can be broken down into the cognitive processes of representing, in which the problem solver builds a coherent mental model of the situation described in the problem; planning, in which the problem solver devises a solution plan; and executing, in which the problem solver carries out the plan. For example, to teach representing skills, Low and Over (1989) gave students practice with feedback in analyzing word problems; for each problem, students had to indicate if it contained enough information, not enough information (and to indicate the needed information), or too much information (and to indicate the extraneous information). Students who received this training showed greater improvements in their problem-solving performance than did students who received no instruction or practice in solving problems. Similarly, Quilici and Mayer (2002) found that students who received instruction and practice in sorting statistics word problems based on their underlying structure performed better on recognizing problem types than did untrained students. Overall, students can learn to use strategies for representing word problems.

Several reviews have shown how it is possible to analyze and teach the underlying cognitive processes required in tasks such as comprehending a passage, writing an essay, solving an arithmetic word problem, answering a scientific question, or explaining an historical event (Mayer, 2003; Pressley & Woloshyn, 1995). Teaching of domain-specific thinking skills represents one of educational psychology's greatest successes (Mayer, 2004b).

In addition to teaching specific cognitive skills, such as how to plan to write an essay, it is also possible to teach metacognitive skills, such as how to coordinate one's cognitive processes, how to adjust one's cognitive processing in light of difficulties, and how to have positive beliefs (Mayer, 1998; Zimmerman & Campillo, 2003). Knowing how to use a cognitive skill, such as how to plan in writing, must be complemented by knowing when and where to use it. For example, Pressley (1990, p. 9) noted, "Good strategy users evaluate whether the strategies they are using are producing progress towards goals they have set for themselves" and also consider "the benefits that follow from using the procedures and the amount of effort required to carry out the procedures." Becoming an effective problem solver requires the development of self-awareness about one's thinking processes and self-regulation of one's thinking processes (Pressley & Schneider, 1997; Pressley & Wharton-McDonald, 1997). Students can develop cognitive and metacognitive skills for problem solving through modeling (Butler & Winne, 1995; Duffy, 2002; Schunk, 1989).

CONCLUSION

Overall, problem solving is an important educational goal. In short, we want students to be able to apply what they learned in new situations. In this chapter, we explored seven methods for promoting learning of problem-solving skill that are supported by research evidence and consistent with cognitive theory: load-reducing methods, such as providing enough practice to build automaticity of basic skills; structure-based methods, such as using concrete manipulatives; schema-activation methods, such as advance organizers and pretraining; generative methods, such as elaborating, summarizing, self-explaining, and questioning; guided discovery methods, in which learners receive guidance while solving problems; modeling methods, such as worked-examples and apprenticeship; and teaching thinking skills in stand-alone courses or within specific subjects.

Each of the seven methods is supported by a robust and growing research base. Although exhaustive literature reviews of the evidence concerning each method are beyond the scope of this chapter, we have provided a few examples of representative research studies.

Each of the seven methods is based on a cognitive theory of learning. The first method works by reducing cognitive load in working memory, which frees up capacity to engage in active cognitive processing. The next five methods work by encouraging active cognitive processing during learning, such as encouraging learners to make connections between existing knowledge and incoming information, encouraging learners to organize incoming information into a coherent structure, and encouraging learners to pay attention to relevant incoming information for further processing. The result is meaningful learning outcomes that are encoded in ways that make them useful in solving new problems. The final method works by helping students build a repertoire of cognitive and metacognitive strategies that can be applied in specific problem-solving situations.

Overall, even since our last review (Mayer & Wittrock, 1996), much progress has been made in achieving one of educational psychology's greatest challenges: helping students become better problem solvers. In service of this goal, three educationally relevant principles that emerge from our review of research on problem solving are as follows:

Domain-specific principle: Rather than attempting to teach general problem-solving heuristics, it is better to teach problem solving skills within specific domains.

Near-transfer principle: Rather than expecting problem-solving skill to be applicable to a wide range of problems, it is better to expect that problem solving skills will be largely restricted with respect to their range of applicability.

Knowledge integration principle: Rather than focusing mainly on teaching of facts and procedures or on teaching concepts and strategies, it is better to integrate teaching of all these kinds of knowledge within guided problem solving tasks.

The study of how to improve students' problem solving performance represents a fruitful and important area of research for educational psychologists.

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