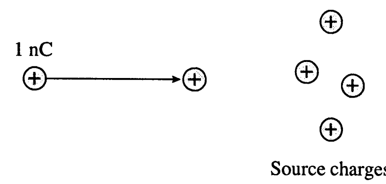
21 Electric Potential

21.1 Electric Potential Energy and Electric Potential

1. A force does 2 μJ of work to push charged particle A toward a set of fixed source charges. Charged particle B has twice the charge of A. How much work must the force do to push B through the same displacement? Explain.

4 μJ. The amount of work is proportional to the charge on which the force acts.

2. A 1 nC charged particle is pushed toward a set of fixed source charges, as shown. In the process, the particle gains 1 μJ of electric potential energy.



   a. How much work was done to push the particle through this displacement? Explain.

   1 μJ. The electric force is conservative.

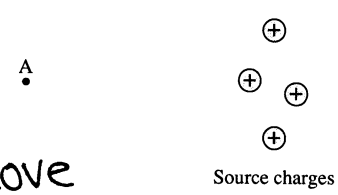
   b. Through what potential difference did the particle move?

   $1000\ V = \frac{1\times10^{-6}\ J}{1\times10^{-9}\ C}$

3. Charged particle A is placed at a point in space where the electric potential is V. Its electric potential energy at that point is U_A. Particle A is removed and replaced by charged particle B, whose potential energy at the same point is U_B. If the charge of B is three times the charge of A, what is the ratio U_B/U_A? Explain.

$\frac{U_B}{U_A} = 3$ The potential energy is proportional to q.

4. Which point, A or B, has the higher electric potential? Why?

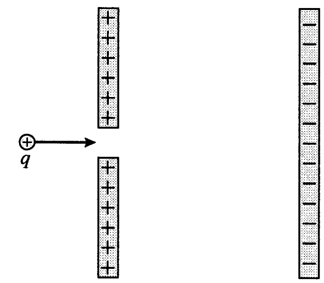


The potential is higher at B. It takes more work to move a positive test charge closer to the source charges at B than further away at A.

21.2 Sources of Electric Potential

21.3 Electric Potential and Conservation of Energy

5. A positive charge q is fired through a small hole in the positive plate of a capacitor. Does q speed up or slow down inside the capacitor? Answer this question twice:



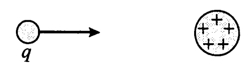
a. First using the concept of force.

It speeds up due to an attractive force to the left.

b. Second using the concept of energy.

It speeds up as it gains kinetic energy because it moves so as to decrease its potential energy.

6. Charge q is fired toward a stationary positive point charge.



a. If q is a positive charge, does it speed up or slow down as it approaches the stationary charge? Answer this question twice:

i. Using the concept of force.

It slows down because the repulsive force it experiences due to the stationary charge is to the left.

ii. Using the concept of energy.

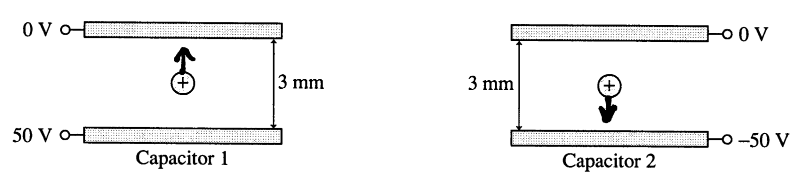
It slows down as it loses kinetic energy because it moves in the direction of increasing potential energy.

b. Repeat part a for q as a negative charge.

i. It speeds up because it feels an attractive force to the right.

ii. It speeds up as it gains kinetic energy because it moves in the direction of decreasing potential energy.

7. The figure shows two capacitors, each with a 3 mm separation. A proton is released from rest in the center of each capacitor.

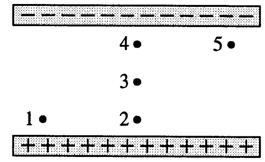


a. Draw an arrow on each proton to show the direction it moves.

b. Which proton reaches a capacitor plate first? Or are they simultaneous? Simultaneous

21.4 Calculating the Electric Potential

8. Rank in order, from largest to smallest, the electric potentials V_1 to V_5 at points 1 to 5.

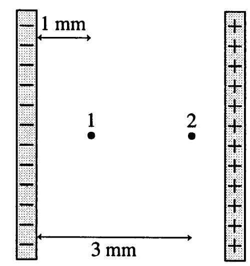


Order: $V_1 = V_2 > V_3 > V_4 = V_5$

Explanation:

The potential decreases as you move away from the positive plate.

9. The figure shows two points inside a capacitor. Let $V = 0$ V at the negative plate.



a. What is the ratio V_2/V_1 of the electric potentials at these two points? Explain.

$V = \frac{Q}{\epsilon_0 A} x$ so $\frac{V_2}{V_1} = \frac{x_2}{x_1} = \frac{3\text{mm}}{1\text{mm}} = 3$

b. What is the ratio E_2/E_1 of the electric field strengths at these two points? Explain.

The field is uniform between the plates.
$E_2/E_1 = 1$

10. A capacitor with plates separated by distance d is charged to a potential difference ΔV_C. All wires and batteries are disconnected, and then the two plates are pulled apart (with insulated handles) to a new separation of distance $2d$.

a. Does the capacitor charge Q change as the separation increases? If so, by what factor? If not, why not?

The capacitor charge remains constant. No charge flows off the capacitor plates so, by conservation of charge, $Q_1 = Q_2$.

b. Does the electric field strength E change as the separation increases? If so, by what factor? If not, why not?

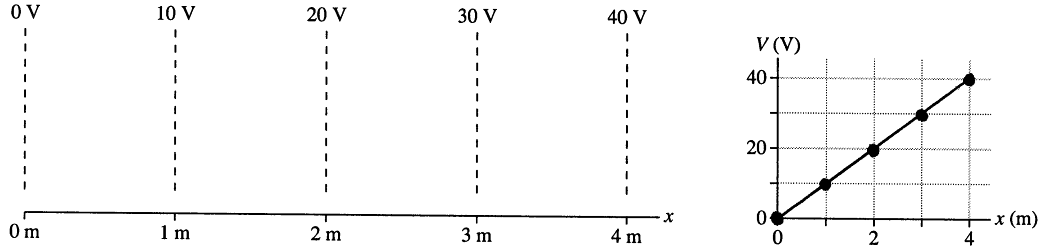
The electric field strength also remains constant.
$E = \frac{\eta}{\epsilon_0} = \frac{Q/A}{\epsilon_0}$ Q, A, and ϵ_0 are all constant.

c. Does the potential difference ΔV_C change as the separation increases? If so, by what factor? If not, why not?

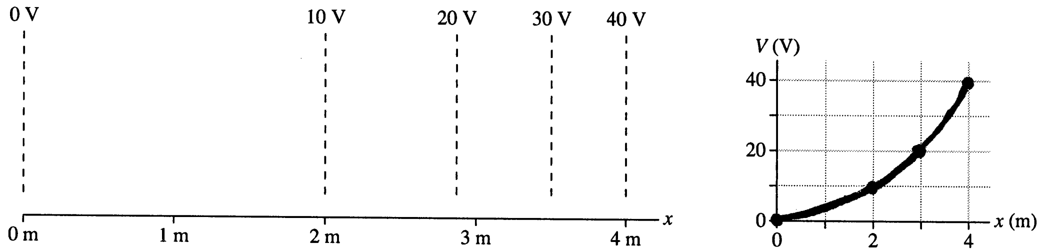
Yes. $\Delta V_C = Ed$ It increases by a factor of 2 because the separation increases by a factor of 2.

11. Each figure shows a contour map on the left and a set of graph axes on the right. Draw a graph of V versus x. Your graph should be a straight line or a smooth curve.

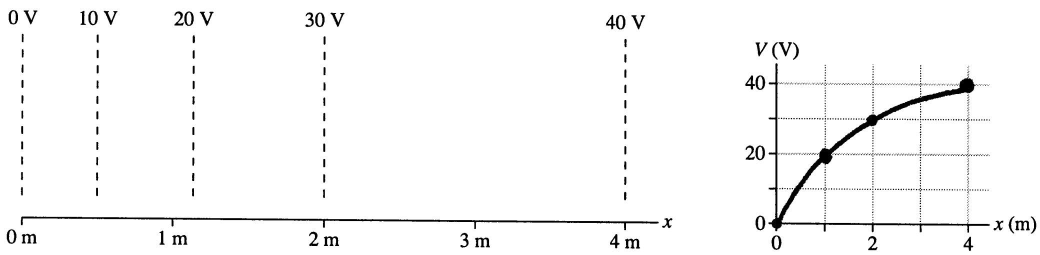
a.



b.

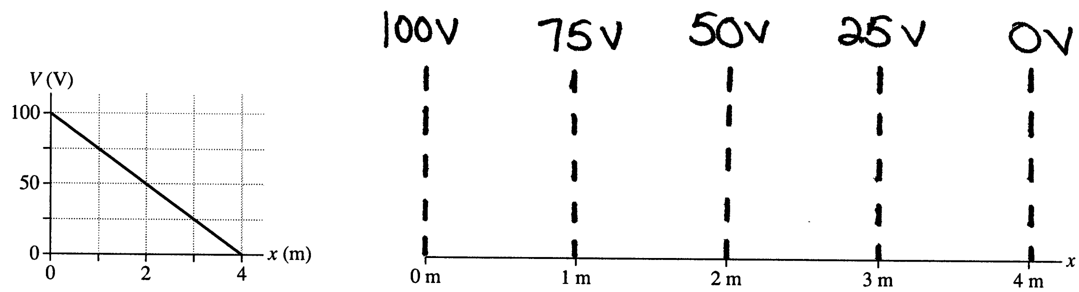


c.

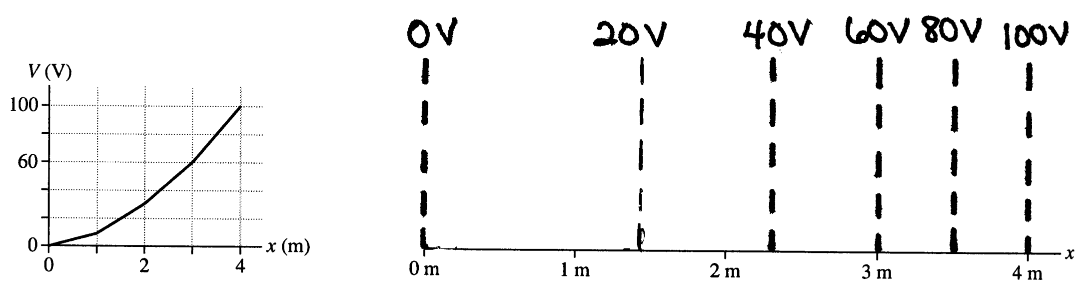


12. Each figure shows a V-versus-x graph on the left and an x-axis on the right. Assume that the potential varies with x but not with y. Draw a contour map of the electric potential. There should be a uniform potential difference between equipotential lines, and each equipotential line should be labeled.

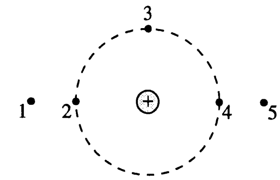
a.



b.



13. Rank in order, from largest to smallest, the electric potentials V_1 to V_5 at points 1 to 5.

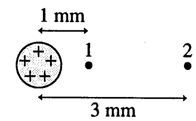


Order: $V_2 = V_3 = V_4 > V_1 = V_5$

Explanation:

$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ As r increases, V increases.

14. The figure shows two points near a positive point charge.



a. What is the ratio V_1/V_2 of the electric potentials at these two points? Explain.

$V \propto \frac{1}{r}$ so $\frac{V_1}{V_2} = \frac{1/1\text{mm}}{1/3\text{mm}} = 3$

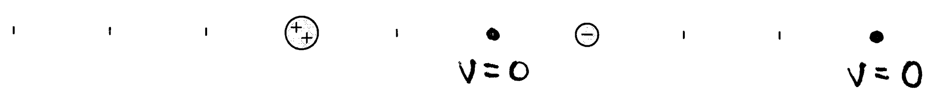
b. What is the ratio E_1/E_2 of the electric field strengths at these two points? Explain.

$E \propto \frac{1}{r^2}$ so $\frac{E_1}{E_2} = \frac{1/(1\text{mm})^2}{1/(3\text{mm})^2} = 9$

15. For each pair of charges below, are there any points (other than at infinity) at which the electric potential is zero? If so, identify them on the figure with a dot and a label. If not, why not?

a. 

No. The potential is always positive because only positive charges are present.

b. 

16. Each figure below shows three points in the vicinity of two point charges. The charges have equal magnitudes. Rank in order, from largest to smallest, the potentials V_1, V_2, and V_3.

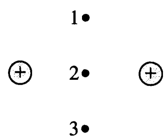
a.

$V_2 > V_1 = V_3$

b.

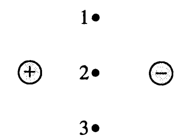
$V_1 > V_2 > V_3$

c.



$V_2 > V_1 = V_3$

d.



$V_1 = V_2 = V_3$

17. An inflatable metal balloon of radius R is charged to a potential of 1000 V. After all wires and batteries are disconnected, the balloon is inflated to a new radius $2R$.

a. Does the potential of the balloon change as it is inflated? If so, by what factor? If not, why not?

Yes. It decreases by a factor of 2.

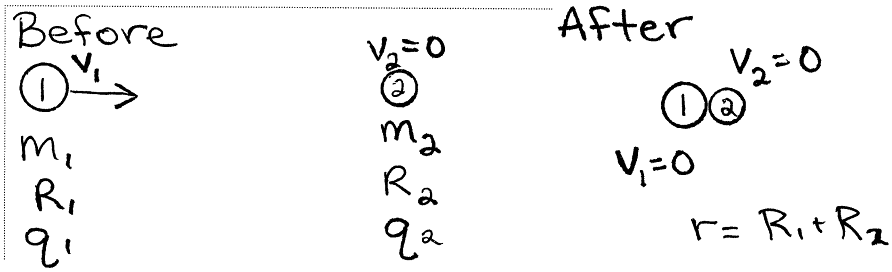
$V_1 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$ $V_2 = \frac{1}{4\pi\epsilon_0}\frac{Q}{(2R)}$ $Q = \text{constant}$

b. Does the potential at a point at distance $r = 4R$ change as the balloon is inflated? If so, by what factor? If not, why not?

No. Outside a sphere the potential is the same as that of a point charge Q located at the center of the sphere. The amount of charge does not change.

18. A small charged sphere of radius R_1, mass m_1, and positive charge q_1 is shot head on with
PSS 21.1 speed v_1 from a long distance away toward a second small sphere having radius R_2, mass m_2, and positive charge q_2. The second sphere is held in a fixed location and cannot move. The spheres repel each other, so sphere 1 will slow as it approaches sphere 2. If v_1 is small, sphere 1 will reach a closest point, reverse direction, and be pushed away by sphere 2. If v_1 is large, sphere 1 will crash into sphere 2. For what speed v_1 does sphere 1 just barely touch sphere 2 as it reverses direction?

a. Begin by drawing a before-and-after visual overview. Initially, the spheres are far apart and sphere 1 is heading toward sphere 2 with speed v_1. The problem ends with the spheres touching. What is speed of sphere 1 at this instant? How far apart are the centers of the spheres at this instant? Label the before and after pictures with complete information—all in symbolic form.



b. Energy is conserved, so we can use Problem-Solving Strategy 21.1. But first we have to identify the "moving charge" q and the "source charge" that creates the potential.

Which is the moving charge? 1 Which is the source charge? 2

c. We're told the charges start "a long distance away" from each other. Based on this statement, what value can you assign to V_i, the potential of the source charge at the initial position of the moving charge? Explain.

$V_i = 0$ $\quad V = \frac{1}{4\pi\epsilon_0}\frac{q_2}{d}$ where d = distance between charges 1 and 2.
d is initially very large.

d. Now write an expression in terms of the symbols defined above (and any constants that are needed) for the initial energy $K_i + qV_i$.

$K_i + qV_i = \frac{1}{2}m_1 v_1^2 + 0$

e. Referring to information on your visual overview, write an expression for the final energy.

$K_f + qV_f = 0 + \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{(R_1+R_2)}$

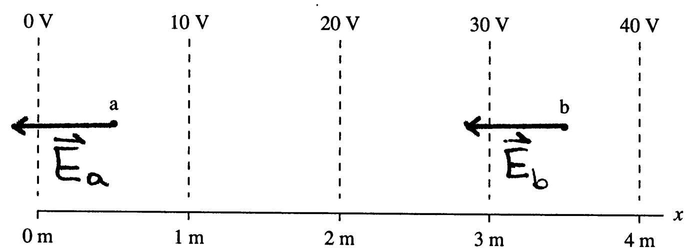
f. Energy is conserved, so finish the problem by solving for v_1.

$$v_1 = \left[\frac{q_1 q_2}{2\pi\epsilon_0 m_1 (R_1+R_2)}\right]^{1/2}$$

21.5 Connecting Potential and Field

19. For each contour map:
    i. Estimate the electric fields $\vec{E}_a$ and $\vec{E}_b$ at points a and b. Don't forget that $\vec{E}$ is a vector. Show how you made your estimate.
    ii. Draw electric field vectors on top of the contour map.

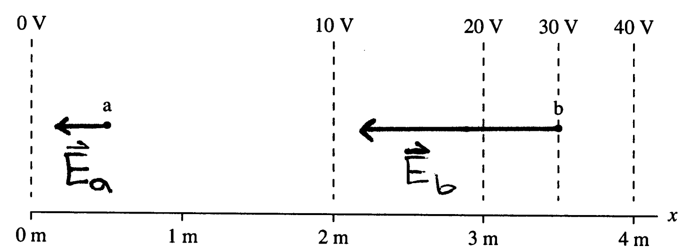
a.



$\vec{E}_a = \frac{-\Delta V}{\Delta x} = \frac{10V}{1m} = 10\frac{V}{m}$

$\vec{E}_b = 10\frac{V}{m}$

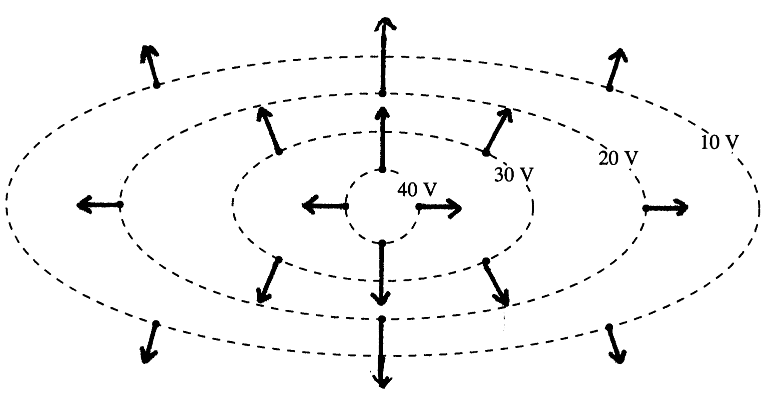
b.



$\vec{E}_a = \frac{10V - 0V}{2m - 0m} = 5\frac{V}{m}$

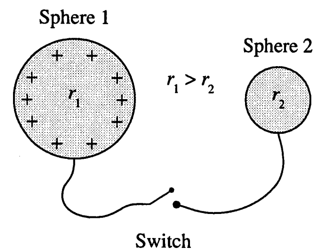
$\vec{E}_b = \frac{40V - 20V}{4m - 3m} = 20\frac{V}{m}$

20. Draw the electric field vectors at the dots on this contour map. The length of each vector should be proportional to the field strength at that point.



The field vectors are perpendicular to the equipotentials.

21. Two metal spheres are connected by a metal wire that has a switch in the middle. Initially, the switch is open. Sphere 1, with the larger radius, is given a positive charge. Sphere 2, with the smaller radius, is neutral. Then the switch is closed. Afterward, sphere 1 has charge Q_1, is at potential V_1, and the electric field strength at its surface is E_1. The values for sphere 2 are Q_2, V_2, and E_2.



a. Is V_1 larger than, smaller than, or equal to V_2? Explain.

$V_1 = V_2$ Both spheres and the wire become one conductor, all at the one same potential.

b. Is Q_1 larger than, smaller than, or equal to Q_2? Explain. $Q_1 > Q_2$ $V_1 = V_2$

$$\frac{1}{4\pi\epsilon_0}\frac{Q_1}{r_1} = \frac{1}{4\pi\epsilon_0}\frac{Q_2}{r_2}; \quad \frac{Q_1}{r_1} = \frac{Q_2}{r_2} \quad r_1 > r_2 \Rightarrow Q_1 > Q_2$$

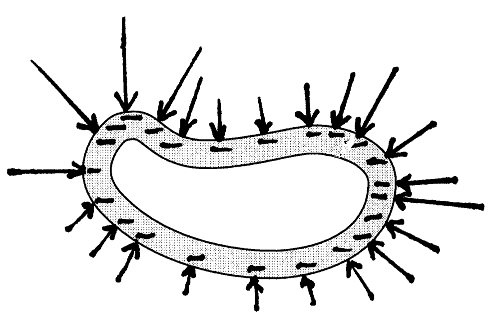
c. Is E_1 larger than, smaller than, or equal to E_2? Explain. $E_1 < E_2$

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{r_1^2} \quad E_2 = \frac{1}{4\pi\epsilon_0}\frac{Q_2}{r_2^2}$$

22. The figure shows a hollow metal shell. A negatively charged rod touches the top of the sphere, transferring charge to the shell. Then the rod is removed.

a. Show on the figure the equilibrium distribution of charge.

b. Draw the electric field diagram.



21.6 The Electrocardiogram

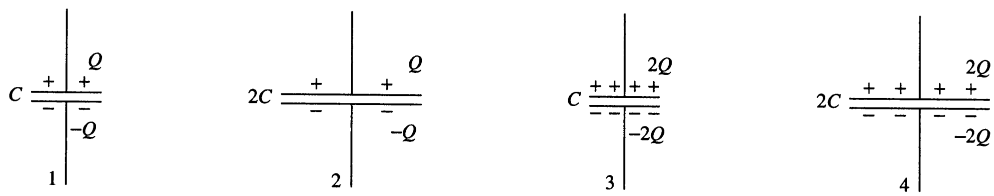
No exercises for this section.

21.7 Capacitance and Capacitors

21.8 Dielectrics and Capacitors

21.9 Energy and Capacitors

23. Rank in order, from largest to smallest, the potential differences $(\Delta V_C)_1$ to $(\Delta V_C)_4$ of these four capacitors.



Order: $(\Delta V_c)_3 > (\Delta V_c)_1 = (\Delta V_c)_4 > (\Delta V_c)_2$

Explanation:
$(\Delta V_c)_1 = \frac{Q}{C}$ $(\Delta V_c)_2 = \frac{Q}{2C} = \frac{1}{2}(\Delta V_c)_1$ $(\Delta V_c)_3 = \frac{2Q}{C} = 2(\Delta V_c)_1$

$(\Delta V_c)_4 = \frac{2Q}{2C} = (\Delta V_c)_1$

24. A parallel-plate capacitor has capacitance C. Suppose all three dimensions of the capacitor are doubled—that is, increased by a factor of 2. By what factor does the capacitance increase? Explain.

The capacitance doubles because the area quadruples but the separation only doubles.

$C_0 = \frac{\varepsilon_0 A}{d} \rightarrow \frac{\varepsilon_0 4A}{2d} = 2C_0$

25. The plates of a parallel-plate capacitor are connected to a battery. If the battery voltage is doubled, by what factor does the energy of the capacitor increase?

$U = \frac{CV^2}{2}$ so doubling the voltage increases the stored energy by 4 times.

26. A parallel-plate capacitor is charged, and then disconnected from the battery that charged it; both plates are now electrically isolated. The capacitor charge, electric field strength, and potential difference are q_i, E_i, and $(\Delta V_C)_i$. Then a dielectric is inserted between the plates. Afterward, the charge, electric field, and potential difference are q_f, E_f, and $(\Delta V_C)_f$.

a. Is q_f larger than, smaller than, or the same as q_i? Explain.

$q_f = q_i$ because it was electrically isolated so the charge has nowhere to go.

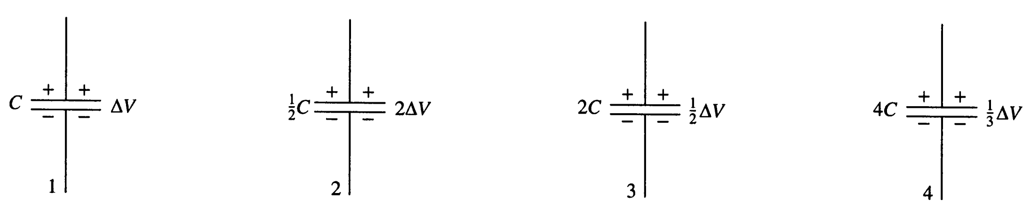
b. Is E_f larger than, smaller than, or the same as E_i? Explain.

$E_f < E_i$ The polarization of the dielectric reduces the field between the plates.

c. Is $(\Delta V_C)_f$ larger than, smaller than, or the same as $(\Delta V_C)_i$? Explain.

$(\Delta V_c)_f < (\Delta V_c)_i$ because the potential here is just the field times the plate separation.

27. Rank in order, from largest to smallest, the energies $(U_C)_1$ to $(U_C)_4$ stored in each of these capacitors.



Order: $(U_c)_2 > (U_c)_1 > (U_c)_3 > (U_c)_4$

Explanation:

$U_1 = \frac{1}{2} C (\Delta V_c)^2$

$U_2 = \frac{1}{2} (\frac{1}{2} C)(2\Delta V_c)^2 = 2U_1$

$U_3 = \frac{1}{2} (2C)(\frac{1}{2}\Delta V_c)^2 = \frac{1}{2} U_1$

$U_4 = \frac{1}{2} (4C)(\frac{1}{3}\Delta V_c)^2 = \frac{4}{9} U_1$