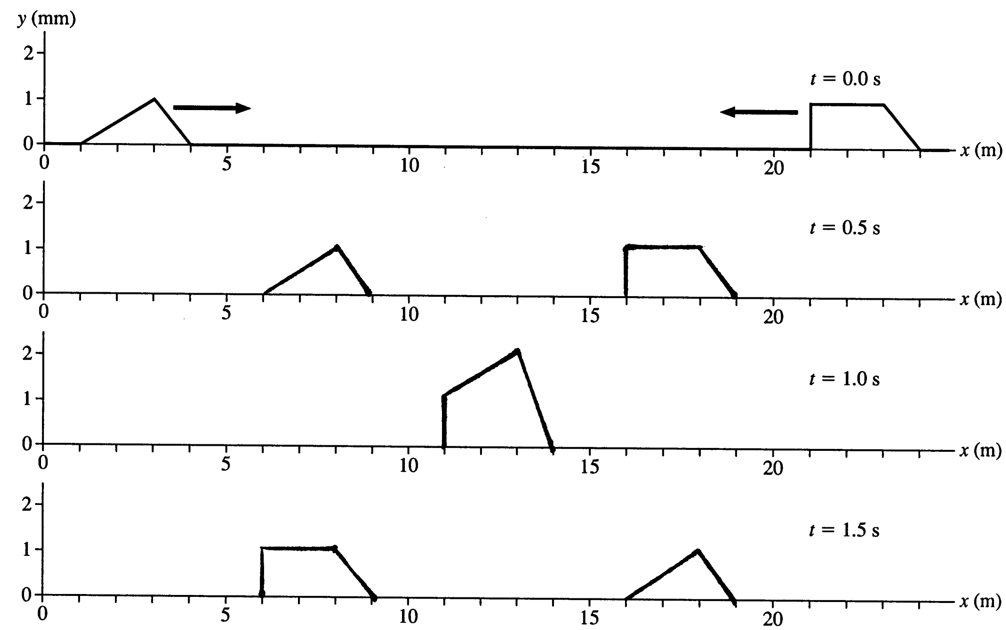
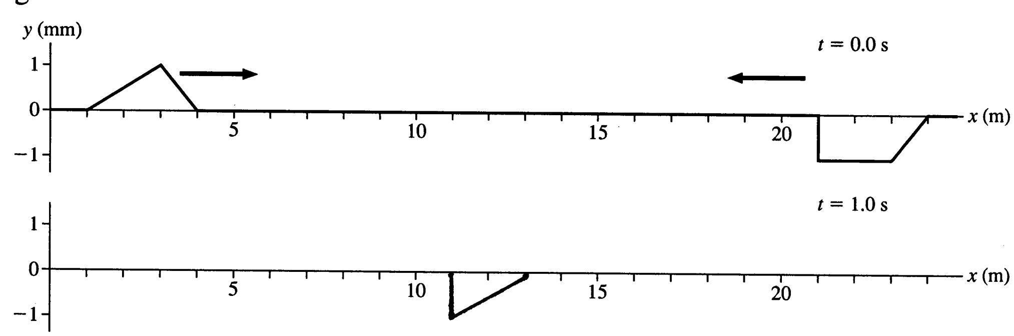
16 Superposition and Standing Waves

16.1 The Principle of Superposition

1. Two pulses on a string are approaching each other at 10 m/s. Draw snapshot graphs of the string at the three times indicated.

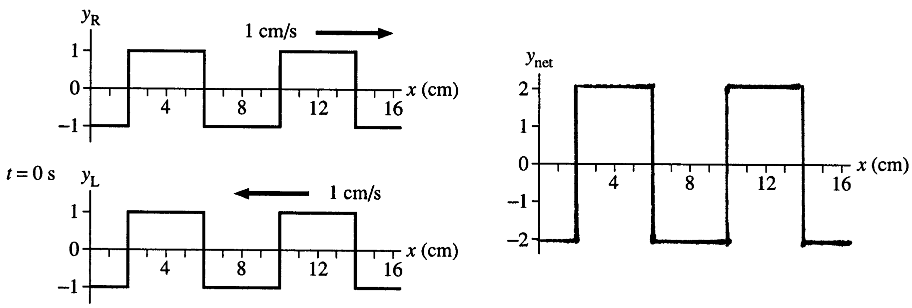


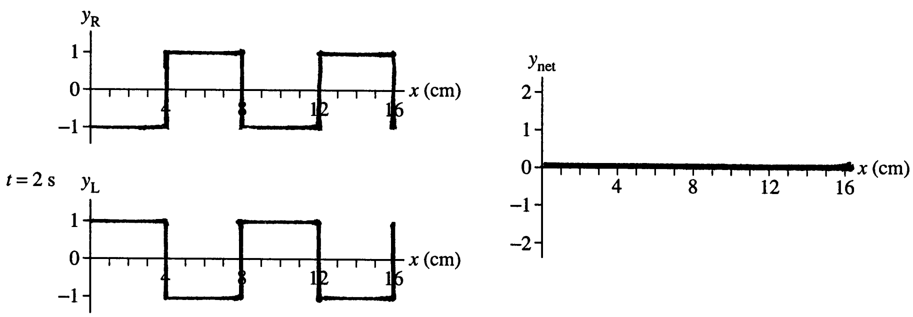
2. Two pulses on a string are approaching each other at 10 m/s. Draw a snapshot graph of the string at $t = 1$ s.

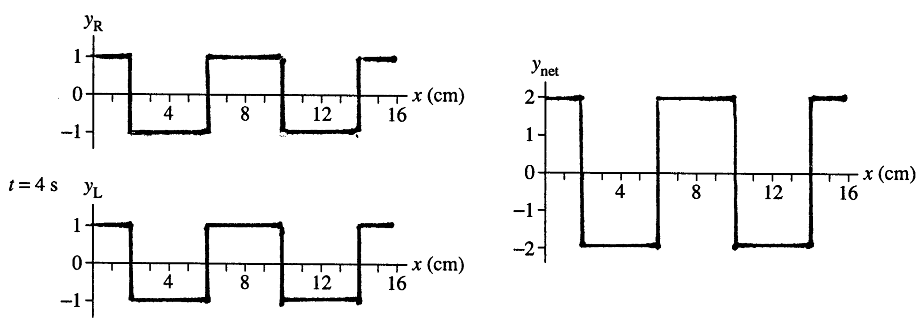


16.2 Standing Waves

3. Two waves are traveling in opposite directions along a string. Each has a speed of 1 cm/s, and an amplitude of 1 cm. The first set of graphs below shows each wave at $t = 0$ s.
   a. On the axes at the right, draw the superposition of these two waves at $t = 0$ s.
   b. On the axes at the left, draw each of the two displacements every 2 s until $t = 8$ s. The waves extend beyond the graph edges, so new pieces of the wave will move in.
   c. On the axes at the right, draw the superposition of the two waves at the same instant.







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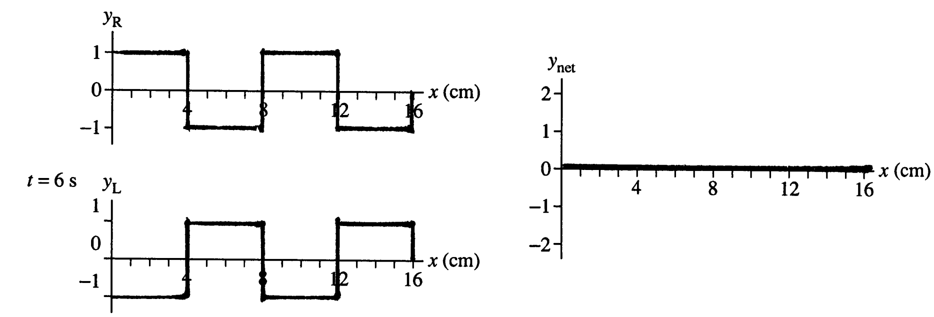
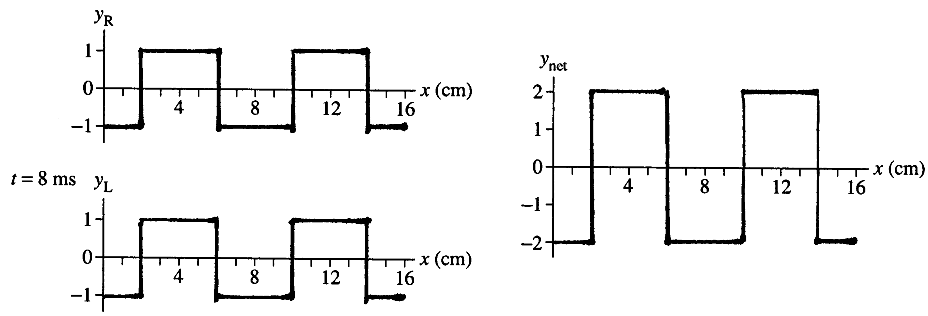
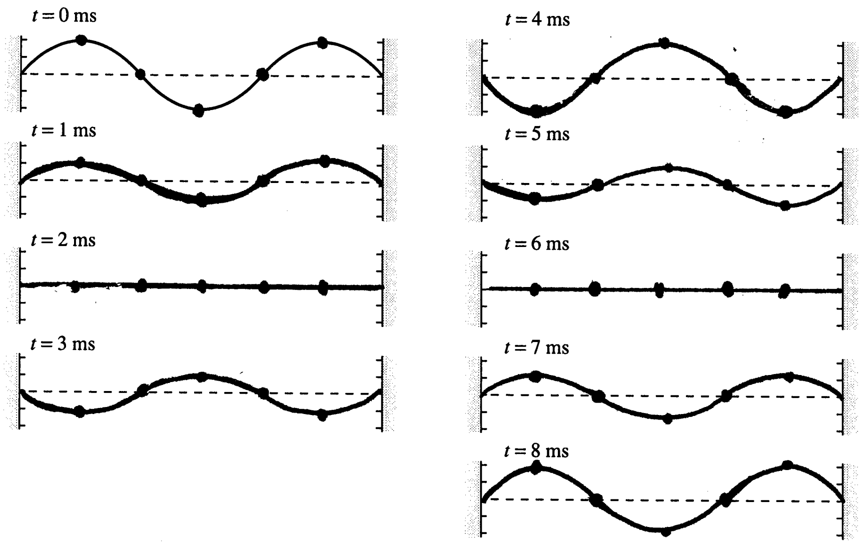

y_R
$t = 6$ s
y_L
y_{net}
x (cm)


y_R
$t = 8$ ms
y_L
y_{net}
x (cm)

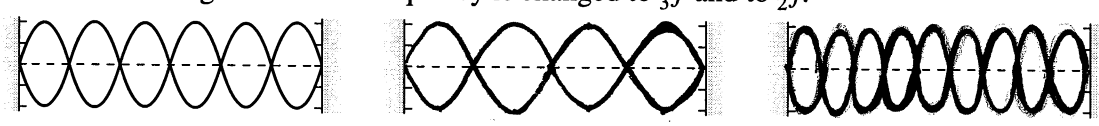
16.3 Standing Waves on a String

4. This standing wave has a period of 8 ms. Draw snapshot graphs of the string every 1 ms from $t = 1$ ms to $t = 8$ ms. Think carefully about the proper amplitude at each instant.



Dots have been added to clarify comparison between graphs.

5. The figure shows a standing wave on a string. It has frequency f.
   a. Draw the standing wave if the frequency is changed to $\frac{2}{3}f$ and to $\frac{3}{2}f$.

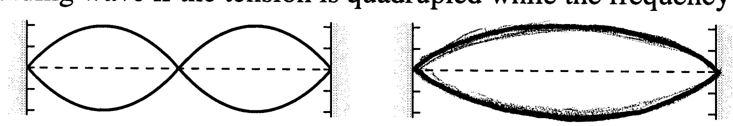


Original wave, frequency f Frequency $\frac{2}{3}f$ Frequency $\frac{3}{2}f$

   b. Is there a standing wave if the frequency is changed to $\frac{1}{4}f$? If so, how many antinodes does it have? If not, why not?

There is no standing wave at $f/4$. At f, $\lambda = \frac{2L}{6}$. Therefore at $f/4$, $\lambda = 4(\frac{2L}{6}) = \frac{4}{3}L$, but this wavelength cannot meet the boundary conditions.

6. The figure shows a standing wave on a string.
   a. Draw the standing wave if the tension is quadrupled while the frequency is held constant.



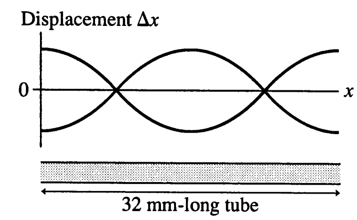
Original wave, tension T Tension $4T$

   b. Suppose the tension is merely doubled while the frequency remains constant. Will there be a standing wave? If so, how many antinodes will it have? If not, why not?

There will be no standing wave because $v \to \sqrt{2}\,v$ and so $\lambda \to \sqrt{2}\,\lambda$, but this will not have nodes at the boundaries.

16.4 Standing Sound Waves

7. The picture shows a *displacement* graph (not a pressure graph) of a standing sound wave in a 32-mm-long tube of air that is open at both ends. That is, the graph shows how far a molecule is displaced from its equilibrium position.



a. Which mode (value of m) standing wave is this? $m = 2$

b. Are the air molecules vibrating vertically or horizontally? Explain.

Horizontally, Sound is a longitudinal wave.

c. At what distances from the left end of the tube do the molecules oscillate with maximum amplitude?

At 0mm, 16mm, and 32mm. (at the antinodes)

8. The purpose of this exercise is to visualize the motion of the air molecules for the standing wave of Exercise 7. On the next page are nine graphs, every one-eighth of a period from $t = 0$ to $t = T$. Each graph represents the displacements at that instant of time of the molecules in a 32-mm-long tube. Positive values are displacements to the right, negative values are displacements to the left.

a. Consider nine air molecules that, in equilibrium, are 4 mm apart and lie along the axis of the tube. The top picture on the right shows these molecules in their equilibrium positions. The dotted lines down the page—spaced 4 mm apart—are reference lines showing the equilibrium positions. Read each graph carefully, then draw nine dots to show the positions of the nine air molecules at each instant of time. The first one, for $t = 0$, has already been done to illustrate the procedure.

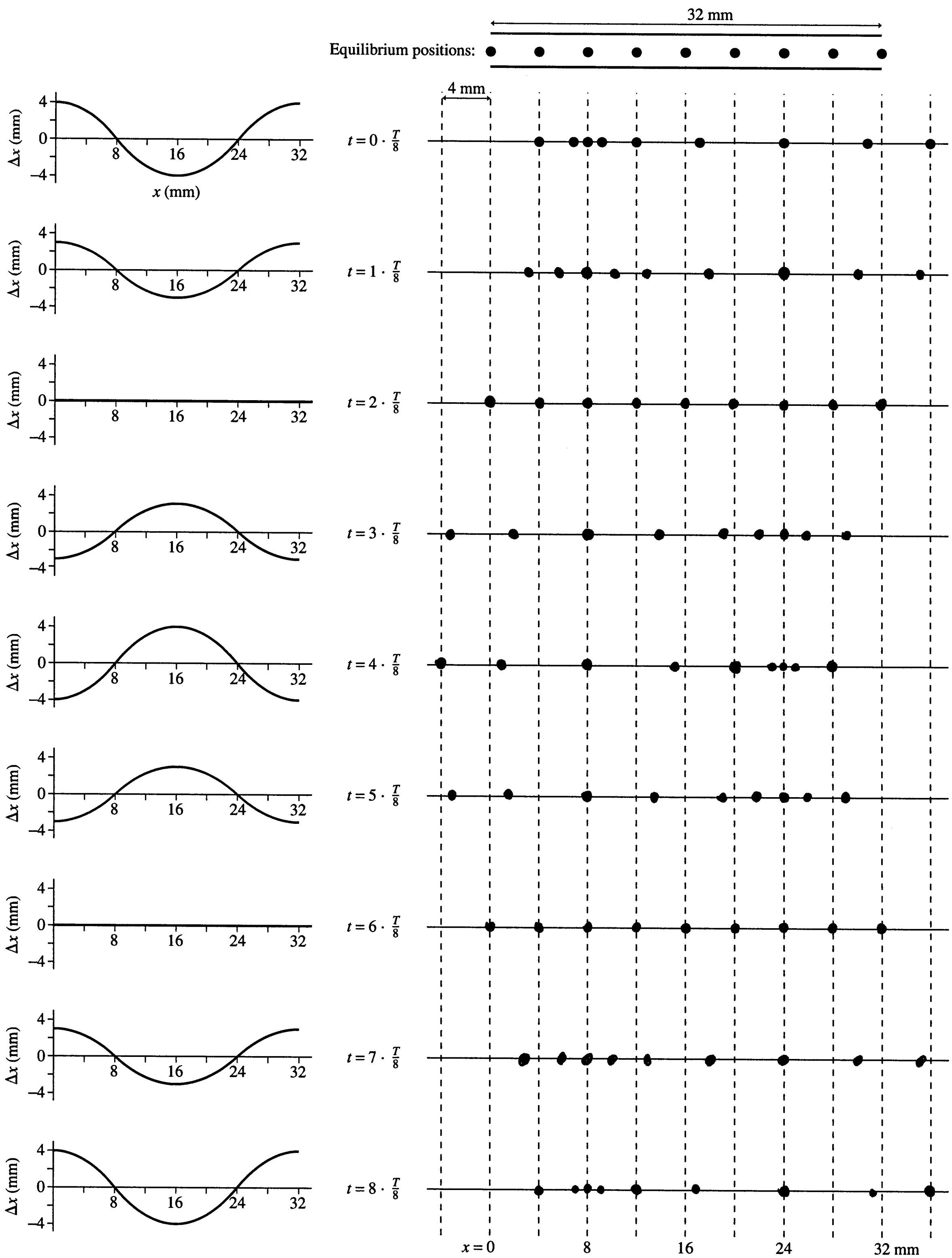
Note: It's a good approximation to assume that the left dot moves in the pattern 4, 3, 0, −3, −4, −3, 0, 3, 4 mm; the second dot in the pattern 3, 2, 0, −2, −3, −2, 0, 2, 3 mm; and so on.

b. At what times does the air reach maximum compression, and where does it occur?

Max compression at time 0, T/2, T   Max compression at position 8mm, 24mm, 8 mm

c. What is the relationship between the positions of maximum compression and the nodes of the standing wave?

The points of maximum compression are nodes.

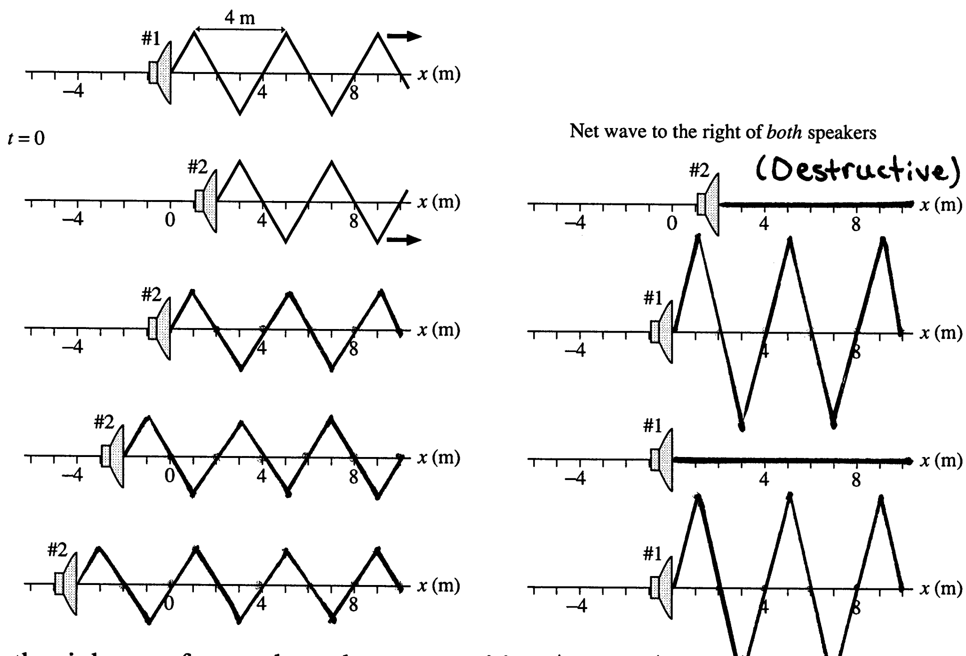

32 mm
Equilibrium positions:
4 mm
Δx (mm)
x (mm)
t = 0 · T/8
t = 1 · T/8
t = 2 · T/8
t = 3 · T/8
t = 4 · T/8
t = 5 · T/8
t = 6 · T/8
t = 7 · T/8
t = 8 · T/8
x = 0
8
16
24
32 mm

16.5 Speech and Hearing

16.6 The Interference of Waves from Two Sources

9. The figure shows a snapshot graph at $t = 0$ s of loudspeakers emitting triangular-shaped sound waves. Speaker 2 can be moved forward or backward along the axis. Both speakers vibrate in phase at the same frequency. The second speaker is drawn below the first, so that the figure is clear, but you want to think of the two waves as overlapped as they travel along the x-axis.

   a. On the left set of axes, draw the $t = 0$ s snapshot graph of the second wave if speaker 2 is placed at each of the positions shown. The first graph, with $x_{speaker} = 2$ m, is already drawn.



   b. On the right set of axes, draw the superposition $\Delta p_{net} = \Delta p_1 + \Delta p_2$ of the waves from the two speakers. Δp_{net} exists only to the right of *both* speakers. It is the net wave traveling to the right.

   c. What separations between the speakers give constructive interference? 0, 4m

   d. What are the $\Delta x/\lambda$ ratios at the points of constructive interference? 0, 1

   e. What separations between the speakers give destructive interference? −2m, +2m

   f. What are the $\Delta x/\lambda$ ratios at the points of destructive interference? ± 1/2

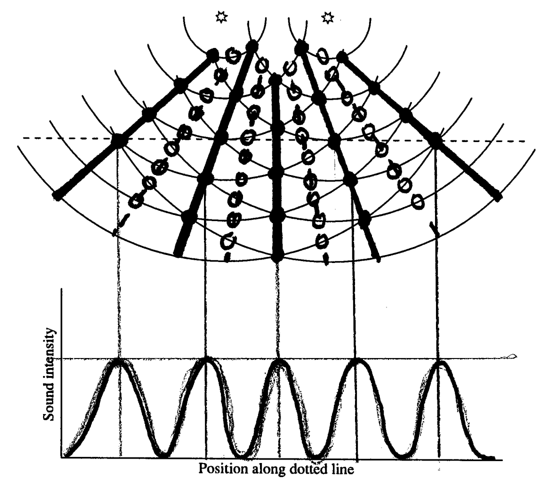
10. The figure shows the wave-front pattern emitted by two loudspeakers.

a. Draw a dot • at points where there is constructive interference. These will be points where two crests overlap *or* two troughs overlap.

b. Draw an open circle ○ at points where there is destructive interference. These will be points where a crest overlaps a trough.

c. Use a **black** line to draw each "ray" of constructive interference. Use a **red** line to draw each "ray" of destructive interference.

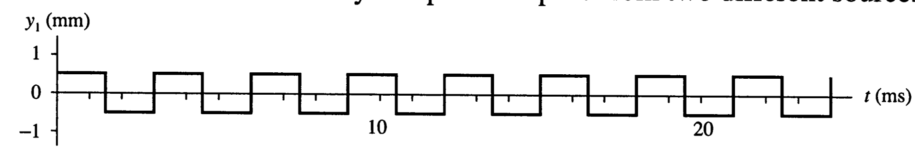
d. Draw a graph on the axes above of the sound intensity you would hear if you walked along the horizontal dotted line. Use the same horizontal scale as the figure so that your graph lines up with the figure above it.

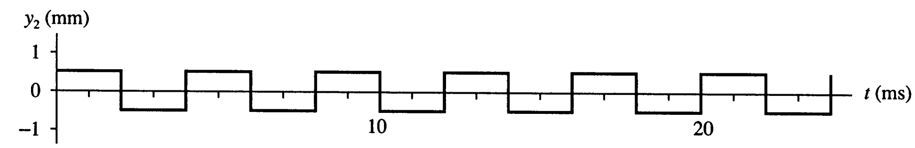


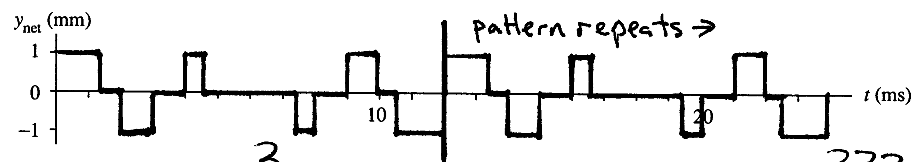
——— Black
– – – Red

16.7 Beats

11. The two waves arrive simultaneously at a point in space from two different sources.







a. Period of wave 1? 3 ms Frequency of wave 1? 333 Hz

b. Period of wave 2? 4 ms Frequency of wave 2? 250 Hz

c. Draw the graph of the net wave at this point on the third set of axes. Be accurate, use a ruler!

d. Period of the net wave? 12 ms Frequency of the net wave? 83.3 Hz

e. Is the frequency of the superposition what you would expect as a beat frequency? Explain.

Yes, the superposition has a frequency equal to the difference in frequency of the two waves.