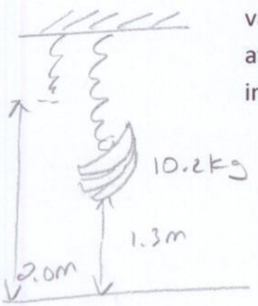
Physics 221
Quiz #2

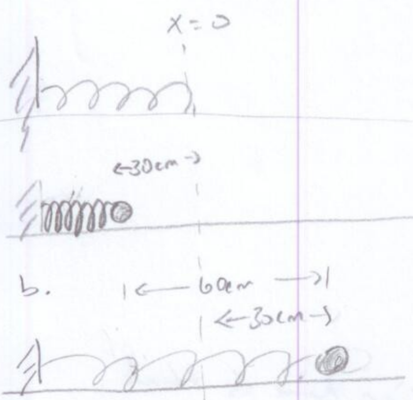
Names: Solution

1. A spring is hanging from the ceiling and you hook a 10.2 kg bunch of bananas to the bottom. The spring was originally hanging 2.0 meters from the floor. After hanging the bananas, the spring stretches so that the end of it is only 1.3 meters from the floor. Your kid brother thinks the only real value for this spring is to use it to shoot things. He takes it and places it on the floor with one end attached to the wall. He then places a rubber ball (200g) on the end of the spring (he pokes the end into the ball so it sticks) and compresses it 30 cm. (the spring and the floor are frictionless) Find
   a. the velocity of the ball when the spring is at its relaxed length.
   b. the velocity of the ball when the spring is 60 cm from where your brother started it.
   c. If the ball didn't stick to the spring, will this be an exciting shooter? Why?
   d. Write the equations of motion for the spring/ball system.
   e. Determine the maximum acceleration of the ball.
   f. What is the ball's period?
   g. What is the speed of the ball at 2.0 cm?
   h. What is the speed of the ball at 2.0 seconds?
   i. Is there anything about this system that you cannot calculate? What is it?



$k = F/x = \frac{10.2\,kg \cdot 9.8\,m/s^2}{0.7\,m}$

$k = 142.8\,N/m$



a. relaxed length means it is neither compressed or stretched so it's at equilibrium which is $x=0$

At this point velocity is a maximum $V_{max} = 2\pi f A$

$f = \frac{1}{2\pi}\sqrt{k/m} = \frac{1}{2\pi}\sqrt{\frac{142.8}{.20\,kg}} = 4.25\,Hz$

$V_{max} = 2\pi\ 4.25\,Hz\ 0.30\,m = \boxed{8.02\,m/s}$

b. 60 cm from where it started (fully compressed) will be 30 cm from equilibrium so at it's max stretch which means the velocity will be zero, it's turning around at this point. $\boxed{v = 0\,m/s}$

c. It would "shoot" at 8.02 m/s which means it'll take a full second to go across a typical living room. That's not terribly exciting.

$F = -kx$ $\qquad a = -\frac{k}{m}x$ $\qquad T = 1/f$

Springs: $T = 2\pi\sqrt{\frac{m}{k}}$ $\qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ $\qquad$ pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$ $\qquad f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$ $\qquad g = 9.8\,m/s^2$

$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$ $\qquad U_g = mgh$ $\qquad K = \frac{1}{2}mv^2$

Equations of motion general: $x = A\cos(2\pi f t)$ $\qquad v = -2\pi f A \sin(2\pi f t)$ $\qquad a = -(2\pi f)^2 A \cos(2\pi f t)$

d. $x = A\cos(2\pi f t)$

$v = -A 2\pi f \sin(2\pi f t)$

$a = -A(2\pi f)^2 \cos(2\pi f t)$

$x = 0.30\,m \cos(2\pi\ 4.25\,Hz\ t)$

$x = 0.30\,m \cos(26.7\tfrac{1}{s} t)$

$v = -8.01\,m/s \sin(26.7\tfrac{1}{s} t)$

$a = -214\,m/s^2 \cos(26.7\tfrac{1}{s} t)$

e. $a_{max} = 214\,m/s^2$

or you can recalculate using $a_{max} = A(2\pi f)^2$

f. $T = 1/f = 1/4.25\,Hz = 0.235\,s$

g. Use eqt's of motion or conservation of energy.

$0.02\,m = 0.30\,m \cos(26.7\,t)$

$0.0667 = \cos(26.7\,t)$

$\cos^{-1}(0.0667) = 26.7\,t$

$\frac{1.5}{26.7} = t = 0.056\,s$

$v = 8.01\,m/s \sin(26.7 \cdot 0.056\,s)$

$= -8.00\,m/s$

Use energy: $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$

$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm 8.0\,m/s$

↑ to choose sign have to think about the direction.

h. $v = 8.01\,m/s \sin(26.7 \cdot 2.0\,s)$

$= 0.28\,m/s$ very small value which will appear to fluctuate depending on your rounding

i. Everything about the mechanical motion is covered by your equations of motion. But the mass of the spring, or composition of the ball or spring, color of the ball, we do not know.