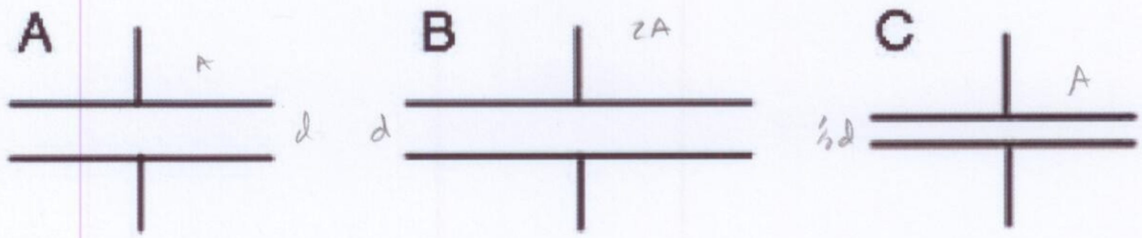
## Comparing capacitors

Three pairs of conducting plates (capacitors) are shown in the figure below. Pair B has the same separation as pair A but twice the plate area. Pair C has the same area as pair A but half the separation between the plates.




If each pair is connected to an identical battery so that the potential difference between each pair of plates is equal to the same value, $\Delta V$, rank the pairs of plates by the magnitude of the electric field between the plates. (Give your answer in the form of a string of relations, like E=F>G=0 using only greater than signs and indicating if any of the answers are equal to each other or to zero.) Explain your reasoning.

$E = \frac{\Delta V}{d}$

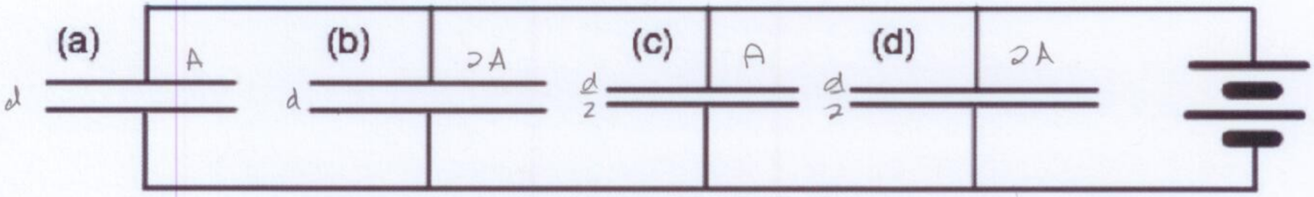
Only the potential difference and the plate spacing affect the value of the Electric Field. This means $E_A = E_B$ but $E_C$ is different. The Electric field is inversely proportional to the plate spacing and capacitor C has a smaller spacing thus a Larger Electric Field.

$\boxed{E_C > E_A = E_B}$

# Comparing capacitors 2

A series of parallel plate capacitors are connected to a battery as shown in the figure below and have the areas and separations of the plates shown in the table at the right.

| Capacitor | Area | Plate Separation |
|---|---|---|
| (a) | A | d |
| (b) | 2A | d |
| (c) | A | d/2 |
| (d) | 2A | d/2 |




A. Rank the capacitors according to the magnitude of the electric field between their plates. Use a ranking such as E>F=G>H=0, that is, only use " >" signs, indicate if any two situations produce equal fields, and indicate if any of the fields are zero.

Each capacitor has the same potential difference because each one has a direct connection to the battery.

$E = -\frac{\Delta V}{d}$

$\boxed{E_c = E_d > E_a = E_b}$

B. Rank the capacitors according to the amount of charge on their top plates. Use a ranking such as E>F=G>H=0, that is, only use " >" signs, indicate if any two situations produce equal charge, and indicate if any of the charges are zero.

$Q = C\Delta V$ so we need to find C first. $C = \frac{\varepsilon_0 A}{d}$ $\boxed{Q_d > Q_b = Q_c > Q_a}$

$Q = \frac{\varepsilon_0 A \Delta V}{d}$ $Q_a = \frac{\varepsilon_0 A \Delta V}{d}$ $Q_b = \frac{\varepsilon_0 2A \Delta V}{d}$ $Q_c = \frac{\varepsilon_0 A \Delta V}{d/2}$ $Q_d = \frac{\varepsilon_0 2A \Delta V}{d/2}$

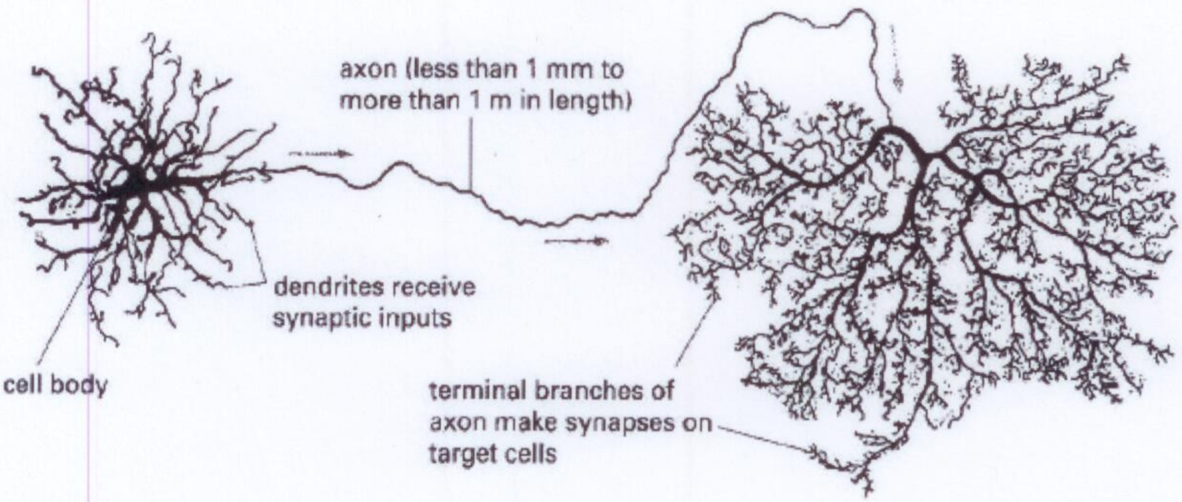
C. Rank the capacitors according to the net (total) amount of charge they contain. Use a ranking such as E>F=G>H=0, that is, only use " >" signs, indicate if any two situations produce equal charge, and indicate if any of the charges are zero.

net charge is zero on all because one plate has $-Q$ and the other has $+Q$.

$\boxed{A = B = C = D = 0}$

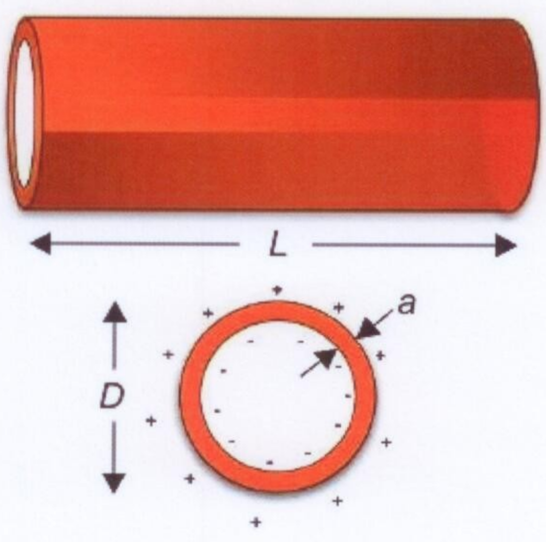
## *Capacitance in nerve cells*

A nerve cell or neuron communicates with other nerve cells through a long "process" (this neuroscience technical term to mean something extended out of a neuron) or cable called an axon. This axon is a long thin tube that is electrically active. A drawing of two neurons connected by one of their axons is shown in the figure below. (From B. Alberts et al., *Molecular Biology of the Cell, Fourth Edition*: Garland Science 2002, p. 1228.)



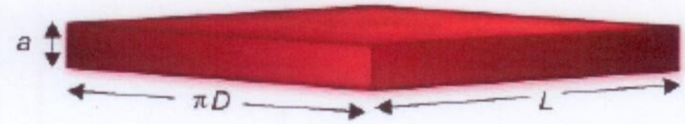

The cell membrane of the axon maintains a potential difference of about 70 mV from the inside to the outside of the membrane. It therefore acts as a capacitor. In this problem we will estimate the capacitance of an axon and the electrical energy stored in the resting axon.

A. We will model the axon membrane as a long thin cylindrical capacitor of thickness *a*, diameter *D*, and length *L*, as shown in the figure at the right above. It is filled with a fluid having a dielectric constant κ (kappa).




If the membrane is thin compared to the radius of the cylinder, we can approximate the cylinder as a parallel plate capacitor by cutting it along the length and flattening it out. It will then look something like shown in the figure at the right below. (We have used that the circumference of the cylinder is $\pi D$.)

Calculate the capacitance of the capacitor in terms of the parameters of its shape and its dielectric constant.




$$C = \frac{\kappa \varepsilon_0 A}{d} \qquad A = \pi D \cdot L \qquad d = a$$

$$\boxed{C = \frac{\kappa \varepsilon_0 \pi D L}{a}}$$

B. The values for the parameters in a typical axon are as follows:

- axon diameter, $D \sim 10\ \mu m = 10^{-5}$ m
- membrane thickness, $a \sim 5$ nm $= 5 \times 10^{-9}$ m
- dielectric constant, $\kappa \sim 7$.

Calculate the capacitance per unit length of an axon in Farads/mm. In addition to getting the value, show that your units work out correctly.

$$C = \frac{7 \cdot 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \cdot \pi \cdot 10^{-5} m \cdot 1 \times 10^{-3} m}{5 \times 10^{-9} m}$$

1mm of Length

so This gives Farads for every 1mm of membrane.

$$\boxed{= 3.89 \times 10^{-10} F/mm}$$

units $= \frac{C^2}{Nm}$ ✓

Farad is $\frac{C}{V} = \frac{C}{Nm/C} = \frac{C^2}{Nm}$ ✓

C. Given the resting voltage difference across the capacitor plates, calculate the amount of electrical energy stored in an axon 1 mm in length.

$\Delta V = 70mV = 70 \times 10^{-3} V$

$U_c = \frac{1}{2} C \Delta V^2 = \frac{1}{2}\, 3.89 \times 10^{-10} F \cdot (70 \times 10^{-3} V)^2 = \boxed{9.5 \times 10^{-13} J}$

## *The power of a nerve*

In the propagation of an electrical pulse along a non-myelinated nerve axon electrical energy stored in the axon's capacitance (i.e., in the separation of charge across the nerve membrane) is discharged and then must be recharged again. Consider a 10 cm axon.

1. The capacitance of an axon per unit length is about 0.3 μF/m and the resting voltage across the membrane is about 70 mV. Estimate the electrical energy required to send a single pulse down the axon.

10cm of axon

$$U_c = \frac{1}{2} C \Delta V^2 = \frac{1}{2}\, 0.3 \times 10^{-6} F (0.1 m)(70 \times 10^{-3} V)^2$$

$$\boxed{= 7.35 \times 10^{-11} J}$$

2. Nerve impulses are short (about 1 ms) and a typical activated rate of sending pulses is 100 pulses/s. Estimate the power in Watts required to maintain the 10 cm of nerve axon activated.

P = Energy/second

$= 1$ pulse $\cdot \frac{100 \text{ pulses}}{\text{second}} \Rightarrow 7.35 \times 10^{-11} J \cdot 100\, \text{Pul}/s = \boxed{7.35 \times 10^{-9} W}$

(Note: The way an axon functions is really more complex than this. We are ignoring the crucial phenomenon of the action potential -- the pulse carrying signals down the axon. Our result just offers an order of magnitude estimate of the energy required.)

RECALL: In units
"m" means milli = $10^{-3}$

"μ" means micro = $10^{-6}$
"n" means nano = $10^{-9}$.

Thus, 750 μF = 750 micro-Farads = 750 x $10^{-6}$ F, etc.

NOTE: 1 F = 1 Farad = (1 Coulomb)/(1 Volt)

## Saving on your electric bill

a. Fluorescent bulbs deliver the same amount of light using much less power. If one kW-hr costs 10¢, estimate the amount of money you would save each month by replacing all the 75 W incandescent bulbs in your house by 15 W fluorescent ones.

I have lights on about 5 hours a day roughly 6 bulbs at a time.

$60W \cdot 6 = 360 W$ · $360W \cdot 5hrs = 1800 Whrs \left(\frac{1kW}{1000W}\right) = 1.8 kWhr$

$1.8 kWhr/day \cdot 30 day = 54 kWhr/month$

$54 kWhr/month \left(\frac{\$0.10}{kWhr}\right) = \boxed{\$5.40/month}$

b. A typical large power plant might generate 1000 Megawatts (1 GW) of electrical power. If all the households in the US were using incandescent bulbs and switched to fluorescent, estimate the number of power plants that would no longer be needed.

I Googled Households in the US. & found 132,312,404 housing units in 2011.

I'll use $1.35 \times 10^8$ as a 2013 estimate.

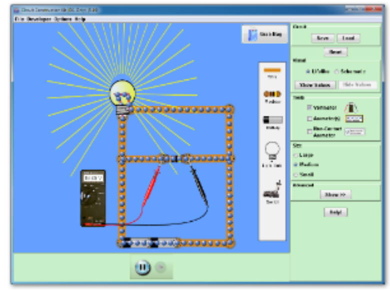
Assuming everyone uses the same lighting I do (25% of those were apartments):

First I need to average out my usage per day since my lights are only on for 5 hours each day but the power plant runs 24 hours a day.

$360W \left(\frac{5hr}{24hr}\right) = 75W$ on average

$75W \cdot 1.35 \times 10^8$ households $\cdot \left[\frac{1 \text{ power plant}}{1000 \times 10^6 W}\right] = \boxed{10 \text{ power plants}}$

**Circuits Lab: Prelab**

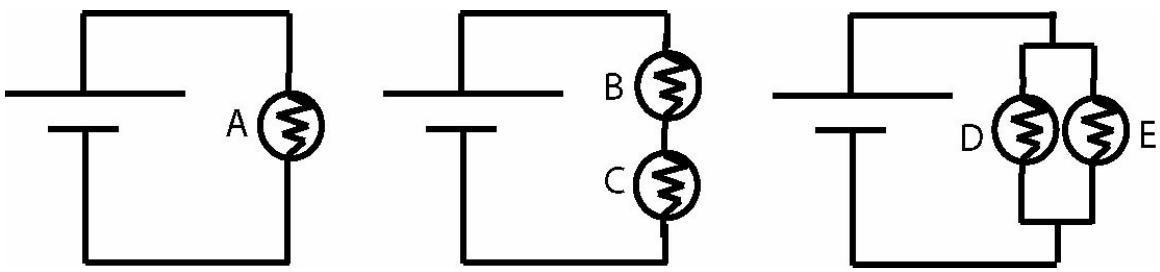


You'll be doing some work with the *Circuit Construction Kit* simulation from PhET.colorado.edu.

Hint: You can right click to disconnect components or to change the values of certain components.

2. Once you've familiarized yourself with the simulation reset the screen. Using a SINGLE light bulb SINGLE battery and SINGLE wire, see if you can get the light bulb to light. Once you're successful, print the result.

3. For the circuits below rank the relative bulb brightness from brightest to dimmest (use CCK if you like). Note all batteries are identical and ideal. All light bulbs are identical and ideal. Bulb brightness reflects the power dissipated in the bulb and that the bulb is a resistor.




A = D = E > B = C

4. In 50 words OR MORE describe WHY the bulbs are ranked as they are. Present your reasoning in every day language so that a friend who has never taken physics would understand your reasoning for why you ranked the bulbs as you did (you can use words like voltage difference, current, energy etc, but no explicit formulas).

The power dissipated by a light bulb depends on two of the three variables: current, potential difference (voltage) and resistance. Since all four of these bulbs have the same resistance and the same battery supplying energy to the circuit, I will use the relationship of power with voltage and resistance. Power is proportional to the square of the voltage and inversely proportional to the resistance.

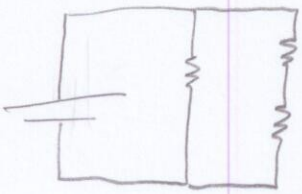
Since resistance is the same for each we just need to figure out how the voltage drops compare. The voltage drop for bulb A is easy since it's the only resistor in the circuit, it's equal to the voltage of the battery.

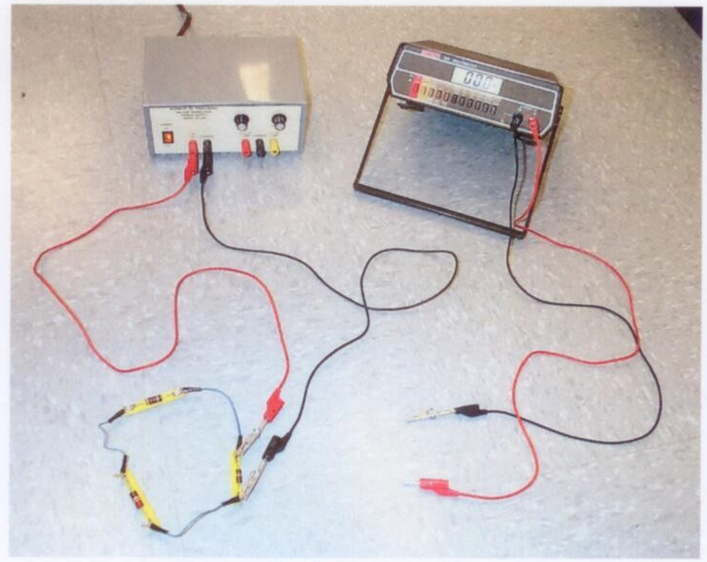
Skip to bulbs D and E. These are also straight forward since each bulb has its own personal direct connection to the battery. That means each bulb also experiences the same voltage drop as the battery.

Bulbs B and C are in "series" so the voltage drop across both of them combined is the voltage of the battery so that means that each bulb has half the voltage drop of the battery.

Therefore: Bulbs A, D and E all have the voltage drop of the battery and they have the same resistance so they must be equally bright. Bulbs B and C each have half the voltage drop of the battery so they must be only a quarter as bright as the other three.

5. Look at the picture below. Draw a *schematic* diagram (like the diagrams in 3.) of the resistors and power supply on the left side of the picture below.





6. Choose the schematic below that you think best represents the circuit in 5. above.