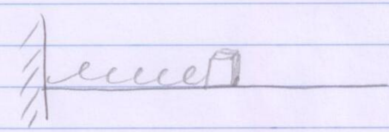
Simple Harmonic Motion (SHM)

SHM is the motion of an object subject to a force that is proportional to the objects displacement.

So far we've study sound waves now springs.



Analyze a spring/mass system

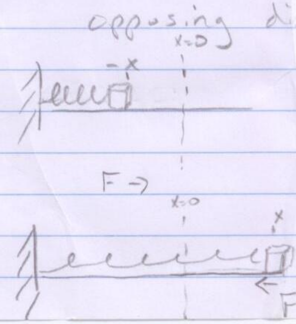
Force
Acceleration
Velocity
Energy

Force $\Rightarrow$ Hooke's Law

$$\boxed{F = -kx}$$

F = Force
k = Spring constant
x = displacement from equilibrium

- Shows it is a restoring force - always opposing displacement (x)



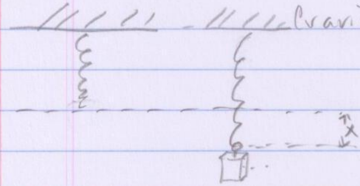
Back to our definition of SHM & see this fits

Force $\propto$ displacement

Spring Constant (k) = Stiffness of the spring
wimpy vs. strong
pen vs. Shocks

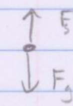
To measure k typically a known force is applied and the displacement (x) is measured.

Gravity is a "known force" FBD



equilibrium

relaxed (not bouncing)

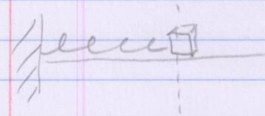


Let's hang 0.55 kg from above spring
measure x to be 4.0 cm

$\Sigma F_y = F_s - F_g = 0$
$= -k(x) - mg = 0$
$= kx = mg$
$k = \frac{mg}{x}$

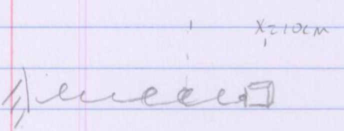
$k = \frac{mg}{x} = \frac{0.55\,kg \cdot 9.8\,m/s^2}{.04\,m} = 135\ N/m$

Now use the same spring horizontally on frictionless surface
$x=0$ $m = 350g$ $k = 135\ N/m$



1. Pull spring 10 cm from equilibrium & let it go.
Find Force & acceleration at
a) $x = 10cm$ & b) $x = 0cm$

$x = 10cm$



a) $F = -kx = -135\ N/m \cdot 0.1\,m = \underline{-13.5\,N}$
$F = ma$ $-13.5\,N = 0.35\,kg\ a$
$a = \underline{-38.6\ m/s^2}$

b) $F = -kx = 0\ N$ $a = 0\ m/s^2$

$$\boxed{a = -\frac{k}{m}x}$$

What is the Amplitude of the oscillation for the spring/mass set up we just analyzed? A = 10 cm

a ranges from $-\frac{k}{m}A$ to $\frac{k}{m}A$

Equations of motion
General form:

$$x = A\cos\left(\frac{2\pi}{T} t\right)$$
$$v = -\frac{2\pi}{T} A \sin\left(\frac{2\pi}{T} t\right)$$
$$a = \left(\frac{2\pi}{T}\right)^2 A \cos\left(\frac{2\pi}{T} t\right)$$

When is $\cos\theta$ Largest? when it = 1

so $a_{max} = \left(\frac{2\pi}{T}\right)^2 A$

just found $a_{max} = k/m\, A$

$$k/m\, \cancel{A} = \left(\frac{2\pi}{T}\right)^2 \cancel{A}$$

so $T = 2\pi\sqrt{m/k}$

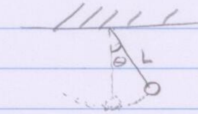
and $f = \frac{1}{2\pi}\sqrt{k/m}$

if we have Period & Amplitude we can write our equations of motion for Springs:

$$x = A\cos\left(\frac{2\pi}{2\pi\sqrt{m/k}}\, t\right)$$

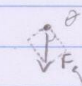
$$\boxed{\begin{aligned} x &= A\cos\left(\sqrt{k/m}\, t\right) \\ v &= -\sqrt{k/m}\, A \sin\left(\sqrt{k/m}\, t\right) \\ a &= -k/m\, A \sin\left(\sqrt{k/m}\, t\right) \end{aligned}}$$

Motion of a pendulum



$F_{tan} = -mg\sin\theta$

FBD



Hooke's law is $F = -kx$

not SHM because $\propto \sin\theta$!

$F_g \sin\theta$ in direction of motion

But if $\theta < 15°$ $\sin\theta \cong \theta$

$F_{Tan} = -mg\theta$

we can turn θ into a displacement s = arclength $= \theta L$

$F_{Tan} = -mg\frac{s}{L} = \frac{-mg}{L}s$ compare to $-kx$

$\frac{mg}{L} = k$

$\boxed{T = 2\pi\sqrt{m/k} \text{ spring}}$

Lab this week

$= 2\pi\sqrt{\frac{m}{mg/L}}$

$\boxed{T = 2\pi\sqrt{L/g} \text{ pendulum}}$ if $\theta < 15°$

Note: Does not depend on mass or Amplitude!

How tall is a tower if a pendulum hanging from the top has a period of 12 s?

$T = 2\pi\sqrt{L/g}$

$\left(\frac{T}{2\pi}\right)^2 g = L$

$\left(\frac{12\,s}{2\pi}\right)^2 9.8\,m/s^2 = \underline{35.8\,m}$