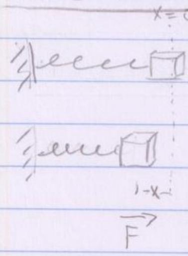
Elastic Potential Energy



Work required to compress/stretch a spring
Average force during compression

$$\bar{F} = \frac{F_0 + F}{2} = \frac{0 + kx}{2} = \frac{1}{2}kx$$

$$W = F \cdot d = F \cdot x = \frac{1}{2}kx^2$$

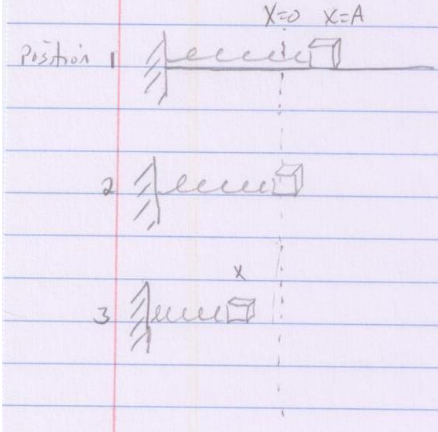
Good test question

$$\boxed{PE_s = \frac{1}{2}kx^2}$$

The energy stored in a compressed or stretched spring (or any other elastic materia) is called elastic potential energy

Conservation of Energy

$$KE_i + PE_{gi} + PE_{si} = KE_f + PE_{gf} + PE_{sf}$$



$E_1 = KE = 0 + PE_g = 0 + PE_s = \frac{1}{2}kA^2$

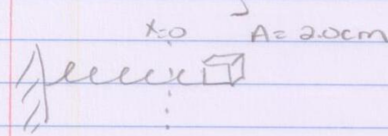
$E_2 = KE = \frac{1}{2}mv^2 + PE_g = 0 + PE_s = 0$

$E_3 = KE = \frac{1}{2}mv^2 + PE_g = 0 + PE_s = \frac{1}{2}kx^2$

$E_1 = E_2 = E_3$ No friction

Back to our previous Spring/mass System

$m = 350g \quad k = 135 N/m$



What is the velocity at
a) 0cm, b) 1.0cm c) -2.0cm?

a) $E_i = E_f$

$E_i = KE_i + PE_{si} = 0 + \frac{1}{2}kA^2 = \frac{1}{2} 135 N/m (0.02m)^2$ (v=0 starts from rest)

$= 0.027 J$

$E_f = \frac{1}{2}mv^2 + 0$ (x=0)

$\frac{1}{2}\, 0.35kg\, v^2 = 0.027J$

$v^2 = \frac{0.027J}{0.175kg}$

$\boxed{v = \pm 0.39 m/s}$

why + or - ?

b) $x = 1.0cm \quad E_i = 0.027J = E_f = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$0.027J = \frac{1}{2}\, 0.35kg\, v^2 + \frac{1}{2} 135 N/m (0.01m)^2$

$0.027J - 0.00675J = 0.175kg\, v^2$

$\frac{0.02025J}{0.175kg} = v^2$

$\boxed{\pm 0.34 m/s = v}$

c) $x = -2.0cm \quad E_i = 0.027J = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$= \frac{1}{2}\, 0.35kg\, v^2 + \frac{1}{2} 135 N/m (0.02m)^2$

$= 0.175kg\, v^2 + 0.027J$

$0.027J - 0.027J = 0.175 v^2$

$\boxed{0 = v}$

It is at the furthest point so v=0 at the instant the mass changes direction

Expression for velocity

$$E_i = E_F$$
$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$
$$\frac{1}{2}kA^2 - \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$
$$k(A^2 - x^2) = mv^2$$
$$\frac{k}{m}(A^2 - x^2) = v^2$$

$$\boxed{v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}}$$

Frictionless

Yesterday we found

$$v = -\sqrt{\frac{k}{m}}\, A \sin\left(\sqrt{\frac{k}{m}}\, t\right)$$

Consider the following problems

1. Find the velocity of a mass when it is 2.0cm from equilibrium
2. Find the velocity of a mass after 2.0 seconds.
3. Find the maximum velocity of the mass.

Which equation(s) would you use for each question?